ICTIR 2011
3rd International Conference on the Theory of Information Retrieval

How to Count  and  ?

User-Rating based Ranking of Items from an Axiomatic Perspective

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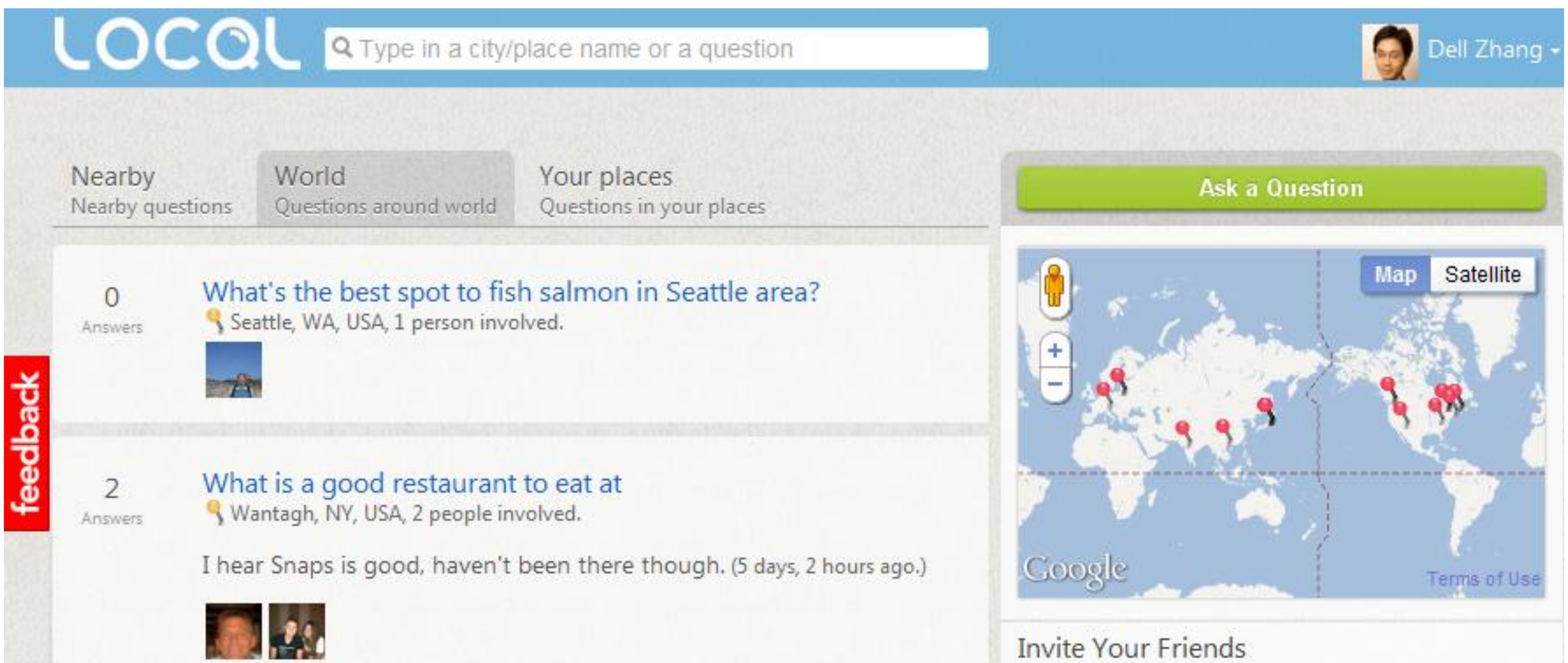
Outline

- Introduction
- Problem
- Popular Methods
- Proposed Approach
- Axiomatic Examination
- Conclusions

Introduction

- Web 2.0
  - Thumb-Up/Thumb-Down



Problem

- User-Rating based Ranking of Items

$$s(n_{\uparrow}, n_{\downarrow}) : \mathbb{N} \times \mathbb{N} \Rightarrow \mathbb{R}$$

Popular Methods

- Difference

$$s(n_\uparrow, n_\downarrow) = n_\uparrow - n_\downarrow$$

Popular Methods

- Difference
  - For example, Urban Dictionary

2. normal — **209** up, **50** down

A word made up by this corrupt society so they could single out and attack those who are different

Normal is nothing but a word made up by society

conformists | worker bees | in crowd | followers | mindless

by Bill Oct 6, 2005 share this add comment

3. normal — **118** up, **25** down

$$s(200, 100) < s(1200, 1000)$$

Popular Methods

- Proportion

$$s(n_\uparrow, n_\downarrow) = \frac{n_\uparrow}{n_\uparrow + n_\downarrow}$$

Popular Methods

- Proportion
  - For example, amazon.com

$$s(200, 1) < s(2, 0)$$

Popular Methods

- Wilson Interval

$$s(n_\uparrow, n_\downarrow) = \frac{\hat{p} + \frac{z^2_{1-\alpha/2}}{2n} - \sqrt{\frac{z^2_{1-\alpha/2}}{n}\left[\hat{p}(1-\hat{p}) + \frac{z^2_{1-\alpha/2}}{4n}\right]}}{1 + \frac{z^2_{1-\alpha/2}}{n}}$$

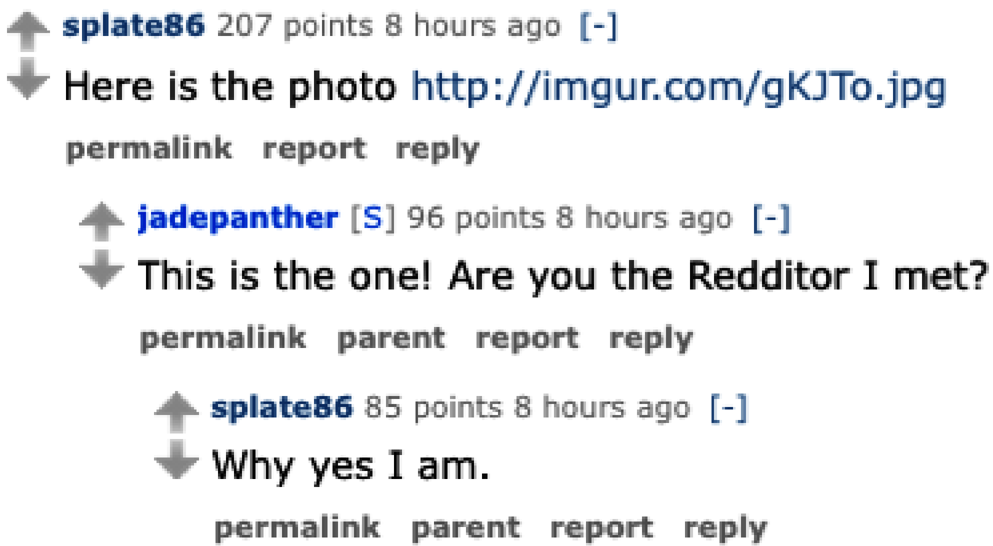
$n = n_\uparrow + n_\downarrow$ $\quad \hat{p} = n_\uparrow / n$ $\quad z_{1-\alpha/2}$

$\alpha = 0.10$

95%

Popular Methods

- Wilson Interval
  - For example, reddit



$$s(1,2) < s(100,200),\ s(5,1) < s(500,501)$$

Proposed Approach

- Information Retrieval
  - Term = User-Rating (↑ / ↓)
  - Document = Item (A Bag of Terms)
  - Query = "↑"

Proposed Approach

- Probability Ranking Principle
  - The proved *optimal* retrieval strategy that minimises the Bayes risk under 1/0 loss

$$\Pr[R = 1 | i, \uparrow]$$

Proposed Approach

- Statistical Language Modelling
  - Unigram Model

$$\Pr[\uparrow | M(i)]$$

Proposed Approach

- Statistical Language Modelling
  - MLE
  - Smoothing
    - Interpolation with a Background Model

Proposed Approach

- Background Model
  - Provided by the prior domain knowledge
    - risk-averse vs risk-loving
  - Estimated from the entire item catalogue

$$p_{\uparrow} = \Pr[\uparrow | M_b] = \frac{\sum_{i=1}^{N} n_{\uparrow}(i)}{\sum_{i=1}^{N}(n_{\uparrow}(i) + n_{\downarrow}(i))}$$

$$p_{\downarrow} = 1 - p_{\uparrow}$$

$$p_{\uparrow} = \Pr[\uparrow | M_b] = \frac{1}{N}\sum_{i=1}^{N} \frac{n_{\uparrow}(i)}{n_{\uparrow}(i) + n_{\downarrow}(i)}$$

Proposed Approach

- Absolute Discounting Smoothing

$$s(n_\uparrow, n_\downarrow) = \Pr[\uparrow | M] = \frac{\max(n_\uparrow - \delta, 0)}{n_\uparrow + n_\downarrow} + \sigma p_\uparrow$$

$$\delta \in [0, 1]$$

$$\sigma = 1 - (\max(n_\uparrow - \delta, 0) + \max(n_\downarrow - \delta, 0))/n$$

Proposed Approach

- Jelinek-Mercer Smoothing

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = (1 - \lambda) \frac{n_{\uparrow}}{n_{\uparrow} + n_{\downarrow}} + \lambda p_{\uparrow}$$

$$\lambda \in [0, 1]$$

Proposed Approach

- Dirichlet Prior Smoothing

$$s(n_\uparrow, n_\downarrow) = \Pr[\uparrow | M] = \frac{n_\uparrow + \mu p_\uparrow}{n_\uparrow + n_\downarrow + \mu}$$

$\mu > 0$

Frequentist => Bayesian

Proposed Approach

- Dirichlet Prior Smoothing
  - Laplace Smoothing
    - $\mu = 2$
    - $p_{\uparrow} = 1/2$

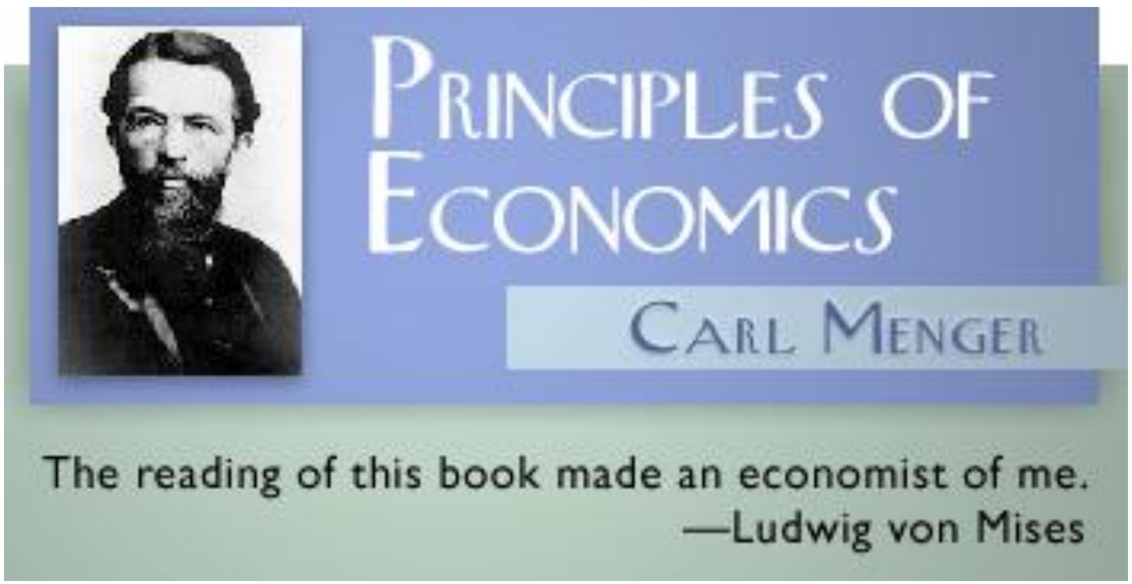
$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = \frac{n_{\uparrow} + 1}{(n_{\uparrow} + 1) + (n_{\downarrow} + 1)}$$

  - Lidstone Smoothing
    - $\mu = 2\epsilon$
    - $p_{\uparrow} = 1/2$

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = \frac{n_{\uparrow} + \epsilon}{(n_{\uparrow} + \epsilon) + (n_{\downarrow} + \epsilon)}$$

Axiomatic Examination

- Two fundamental principles in **Economics** developed by Carl Menger



The paradox of water and diamonds

Axiomatic Examination

- Marginal Utility

$$\Delta_{\uparrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) = s(n_{\uparrow} + 1, n_{\downarrow}) - s(n_{\uparrow}, n_{\downarrow})$$

$$\Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) = s(n_{\uparrow}, n_{\downarrow}) - s(n_{\uparrow}, n_{\downarrow} + 1)$$

Axiomatic Examination

- The Law of Increasing Total Utility

$$\Delta_{\uparrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > 0$$

$$\Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > 0$$

- The Law of Diminishing Marginal Utility

$$\Delta_{\uparrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > \Delta_{\uparrow}^{(s)}(n_{\uparrow} + 1, n_{\downarrow})$$

$$\Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > \Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow} + 1)$$

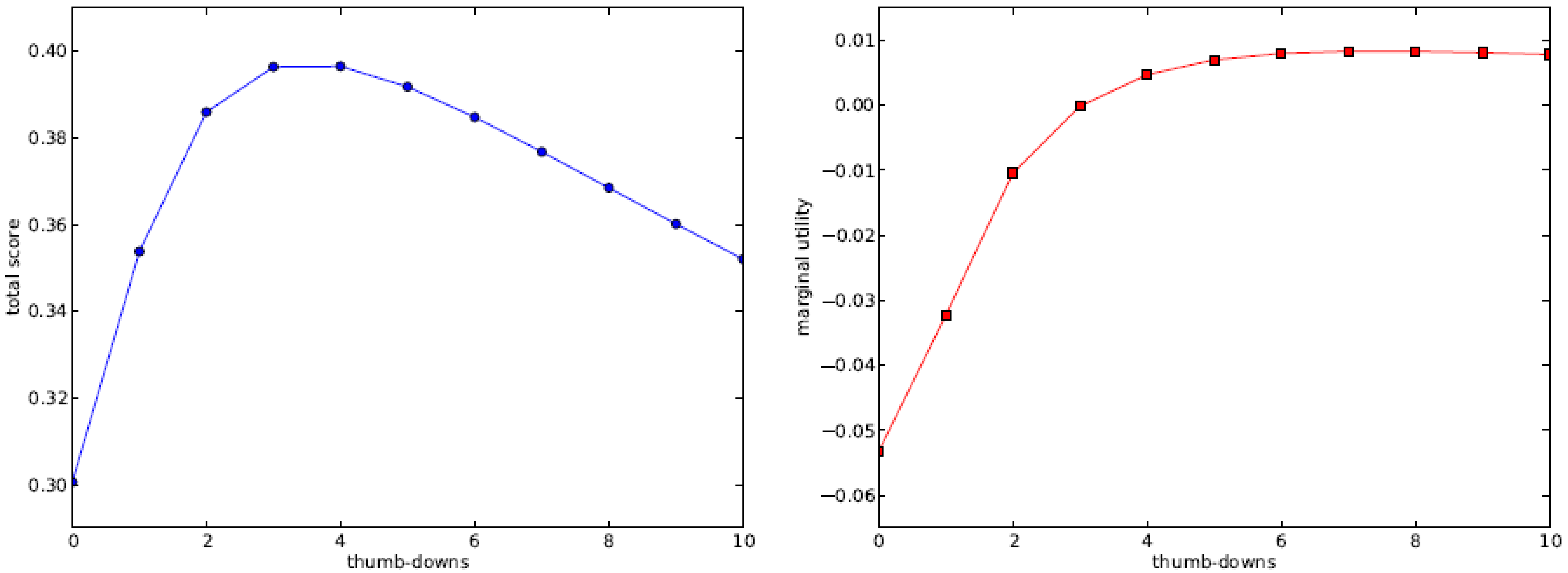
Axiomatic Examination

- Difference
  - Axiom 1; ~~Axiom 2~~.
- Proportion
  - ~~Axiom 1~~; ~~Axiom 2~~.
- Absolute Discounting
  - ~~Axiom 1~~; ~~Axiom 2~~.
- Jelinek-Mercer
  - ~~Axiom 1~~; ~~Axiom 2~~.

$$n_{\downarrow} = 0$$

Axiomatic Examination

- **Proposition**. The Wilson Interval method violates both Axiom 1 and Axiom 2.



The Wilson interval $s(n_{\uparrow}, n_{\downarrow})$ with $n_{\uparrow} = 1$.

Axiomatic Examination

- **Theorem**. The Dirichlet Prior smoothing method satisfies both Axiom 1 and Axiom 2.
  - Corollary 1. The Laplace smoothing method …
  - Corollary 2. The Lidstone smoothing method …

Axiomatic Examination

	Increasing Total Utility	Diminishing Marginal Utility
Difference	$\surd$	$\times$
Proportion	$\times$	$\times$
Wilson Interval	$\times$	$\times$
Absolute Discounting	$\times$	$\times$
Jelinek-Mercer	$\times$	$\times$
Dirichlet Prior	$\surd$	$\surd$

Conclusions

- Contribution
  - An Information Retrieval Approach to User-Rating based Ranking of Items
    - Probability Ranking Principle
    - Statistical Language Modelling
  - An Axiomatic Examination of the Existing and Proposed Methods
    - Increasing Total Utility
    - Decreasing Marginal Utility

Dirichlet Prior smoothing

Conclusions

- Generalisations
  - Graded Ratings
    - => Multiple Thumb-Ups and Thumb-Downs
      - In a 5-star system: *** = 3↑ + 2↓
    - Learning the Weights from Click-Through Data
  - Ageing of User-Ratings
    - Time-Sensitive Language Modelling (ICTIR-2009)

Question Time

(?_?)

Thank You

(^_^)