

# How to Count and ?

User-Rating based Ranking of Items  
from an Axiomatic Perspective

Dell Zhang



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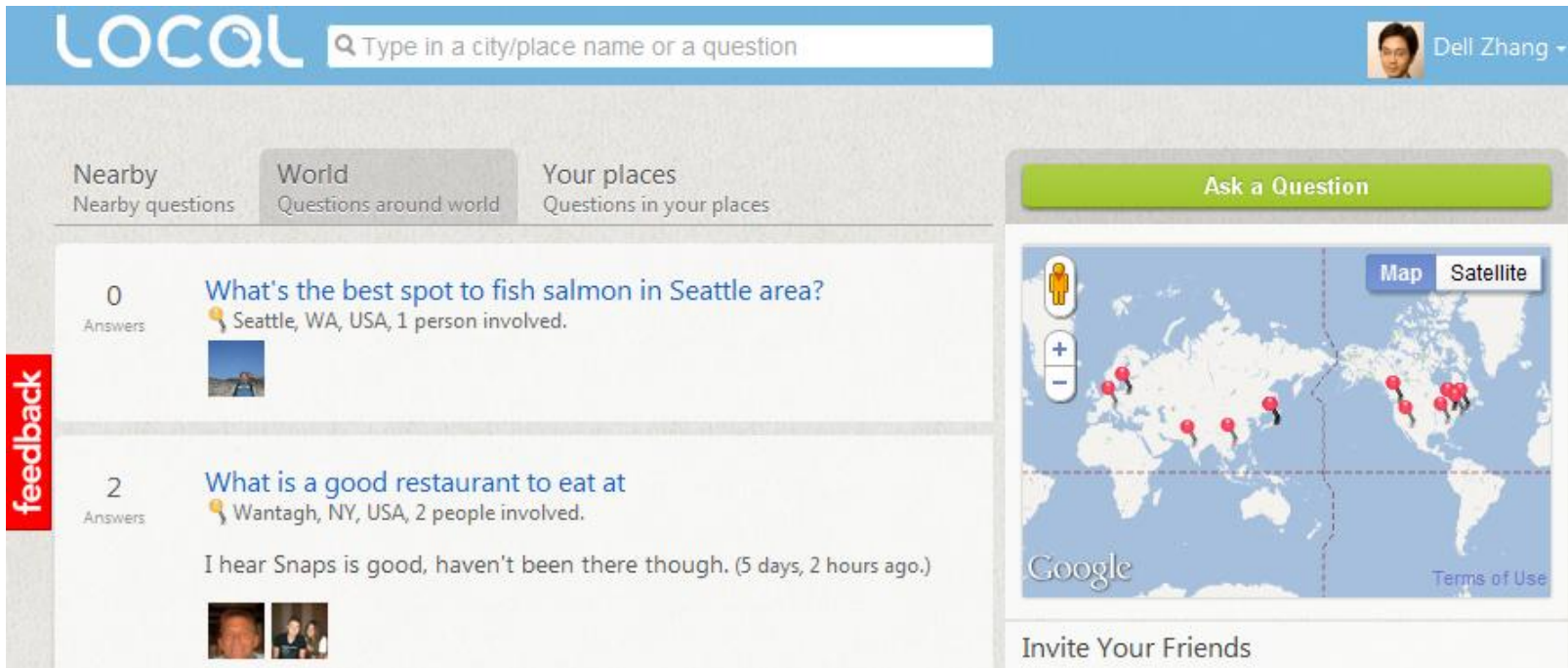
# Outline

- Introduction
- Problem
- Popular Methods
- Proposed Approach
- Axiomatic Examination
- Conclusions

# Introduction

- Web 2.0

- Thumb-Up/Thumb-Down



The screenshot displays the LOCL website interface. At the top, there is a blue header with the LOCL logo and a search bar containing the text "Type in a city/place name or a question". To the right of the search bar is a user profile for "Dell Zhang". Below the header, there are three navigation tabs: "Nearby" (Nearby questions), "World" (Questions around world), and "Your places" (Questions in your places). The main content area shows a list of questions. The first question is "What's the best spot to fish salmon in Seattle area?" with 0 answers and a location tag for Seattle, WA, USA, 1 person involved. The second question is "What is a good restaurant to eat at" with 2 answers and a location tag for Wantagh, NY, USA, 2 people involved. Below the second question, there is a text response: "I hear Snaps is good, haven't been there though. (5 days, 2 hours ago.)" and two small profile pictures. On the right side, there is a green "Ask a Question" button above a world map with several red location pins. The map includes a "Map" and "Satellite" toggle, a person icon, and zoom controls. The Google logo and "Terms of Use" link are visible at the bottom of the map. A vertical red "feedback" button is located on the left side of the question list. At the bottom right, there is an "Invite Your Friends" button.

# Problem

- User-Rating based Ranking of Items

$$s(n_{\uparrow}, n_{\downarrow}) : \mathbb{N} \times \mathbb{N} \Rightarrow \mathbb{R}$$

# Popular Methods



- Difference

$$s(n_{\uparrow}, n_{\downarrow}) = n_{\uparrow} - n_{\downarrow}$$

# Popular Methods

- Difference

– For example, 



**2. normal** 209 up, 50 down  

A word made up by this corrupt society so they could single out and attack those who are different

*Normal is nothing but a word made up by society*

[conformists](#) [worker bees](#) [in crowd](#) [followers](#) [mindless](#)

by [Bill](#) Oct 6, 2005 [share this](#) [add comment](#)

**3. normal** 118 up, 25 down  

$$s(200, 100) < s(1200, 1000)$$

# Popular Methods

- Proportion

$$s(n_{\uparrow}, n_{\downarrow}) = \frac{n_{\uparrow}}{n_{\uparrow} + n_{\downarrow}}$$

# Popular Methods

- Proportion
  - For example, [amazon.com](https://www.amazon.com)

13.



**SALTON HOUSEWARES, INC.  
TR2500C ULTIMATE PLUS  
BREAKMAKER**

Buy new: **\$135.99**

In Stock

★★★★★ (1)

14.



**KitchenAid KP26M1XLC  
Professional 600 Series 6-Quart  
Stand Mixer, Licorice**

Buy new: ~~\$499.99~~ **\$329.99**

10 Used & new from **\$325.00**

★★★★★ (580)

$$s(200, 1) < s(2, 0)$$



# Popular Methods

- Wilson Interval

$$s(n_{\uparrow}, n_{\downarrow}) = \frac{\hat{p} + \frac{z_{1-\alpha/2}^2}{2n} - \sqrt{\frac{z_{1-\alpha/2}^2}{n} \left[ \hat{p}(1 - \hat{p}) + \frac{z_{1-\alpha/2}^2}{4n} \right]}}{1 + \frac{z_{1-\alpha/2}^2}{n}}$$

$$n = n_{\uparrow} + n_{\downarrow}$$

$$\hat{p} = n_{\uparrow}/n$$

$$z_{1-\alpha/2}$$

$$\alpha = 0.10$$

$$95\%$$

# Popular Methods

- Wilson Interval
  - For example,  reddit

↑ **splate86** 207 points 8 hours ago [-]  
↓ Here is the photo <http://imgur.com/gKJTo.jpg>  
permalink report reply

↑ **jadepanther** [S] 96 points 8 hours ago [-]  
↓ This is the one! Are you the Redditor I met?  
permalink parent report reply

↑ **splate86** 85 points 8 hours ago [-]  
↓ Why yes I am.  
permalink parent report reply

$$s(1, 2) < s(100, 200), s(5, 1) < s(500, 501)$$

# Proposed Approach

- Information Retrieval
  - Term = User-Rating ( $\uparrow$  /  $\downarrow$ )
  - Document = Item (A Bag of Terms)
  - Query = “ $\uparrow$ ”

# Proposed Approach

- Probability Ranking Principle
  - The proved *optimal* retrieval strategy that minimises the Bayes risk under 1/0 loss

$$\Pr[R = 1 | i, \uparrow]$$

# Proposed Approach

- Statistical Language Modelling
  - Unigram Model

$$\Pr[\uparrow | M(i)]$$

# Proposed Approach

- Statistical Language Modelling
  - MLE
  - Smoothing
    - Interpolation with a Background Model

# Proposed Approach

- Background Model
  - Provided by the prior domain knowledge
    - risk-averse vs risk-loving
  - Estimated from the entire item catalogue

$$p_{\uparrow} = \Pr[\uparrow | M_b] = \frac{\sum_{i=1}^N n_{\uparrow}(i)}{\sum_{i=1}^N (n_{\uparrow}(i) + n_{\downarrow}(i))}$$

$$p_{\uparrow} = \Pr[\uparrow | M_b] = \frac{1}{N} \sum_{i=1}^N \frac{n_{\uparrow}(i)}{n_{\uparrow}(i) + n_{\downarrow}(i)}$$

$$p_{\downarrow} = 1 - p_{\uparrow}$$

# Proposed Approach

- Absolute Discounting Smoothing

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = \frac{\max(n_{\uparrow} - \delta, 0)}{n_{\uparrow} + n_{\downarrow}} + \sigma p_{\uparrow}$$

$$\delta \in [0, 1]$$

$$\sigma = 1 - (\max(n_{\uparrow} - \delta, 0) + \max(n_{\downarrow} - \delta, 0)) / n$$



# Proposed Approach

- Jelinek-Mercer Smoothing

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = (1 - \lambda) \frac{n_{\uparrow}}{n_{\uparrow} + n_{\downarrow}} + \lambda p_{\uparrow}$$

$$\lambda \in [0, 1]$$

# Proposed Approach

- Dirichlet Prior Smoothing

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = \frac{n_{\uparrow} + \mu p_{\uparrow}}{n_{\uparrow} + n_{\downarrow} + \mu}$$

$$\mu > 0$$

Frequentist => Bayesian

# Proposed Approach

- Dirichlet Prior Smoothing

- Laplace Smoothing

- $\mu = 2$
    - $p_{\uparrow} = 1/2$

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = \frac{n_{\uparrow} + 1}{(n_{\uparrow} + 1) + (n_{\downarrow} + 1)}$$

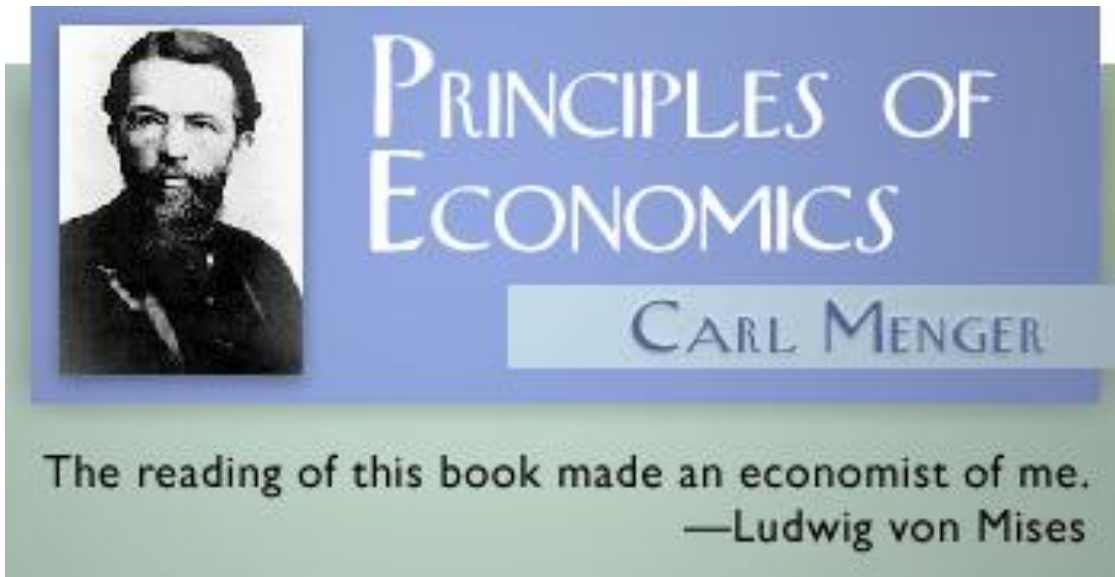
- Lidstone Smoothing

- $\mu = 2\epsilon$
    - $p_{\uparrow} = 1/2$

$$s(n_{\uparrow}, n_{\downarrow}) = \Pr[\uparrow | M] = \frac{n_{\uparrow} + \epsilon}{(n_{\uparrow} + \epsilon) + (n_{\downarrow} + \epsilon)}$$

# Axiomatic Examination

- Two fundamental principles in **Economics** developed by Carl Menger



The paradox of water and diamonds

# Axiomatic Examination

- Marginal Utility

$$\Delta_{\uparrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) = s(n_{\uparrow} + 1, n_{\downarrow}) - s(n_{\uparrow}, n_{\downarrow})$$

$$\Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) = s(n_{\uparrow}, n_{\downarrow}) - s(n_{\uparrow}, n_{\downarrow} + 1)$$

# Axiomatic Examination

- The Law of Increasing Total Utility

$$\Delta_{\uparrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > 0$$

$$\Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > 0$$

- The Law of Diminishing Marginal Utility

$$\Delta_{\uparrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > \Delta_{\uparrow}^{(s)}(n_{\uparrow} + 1, n_{\downarrow})$$

$$\Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow}) > \Delta_{\downarrow}^{(s)}(n_{\uparrow}, n_{\downarrow} + 1)$$

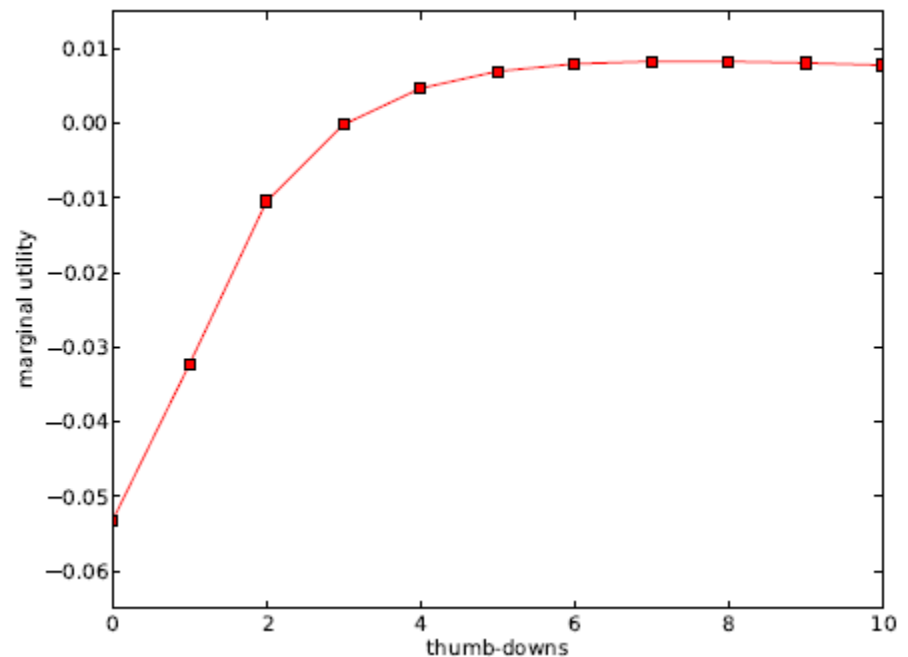
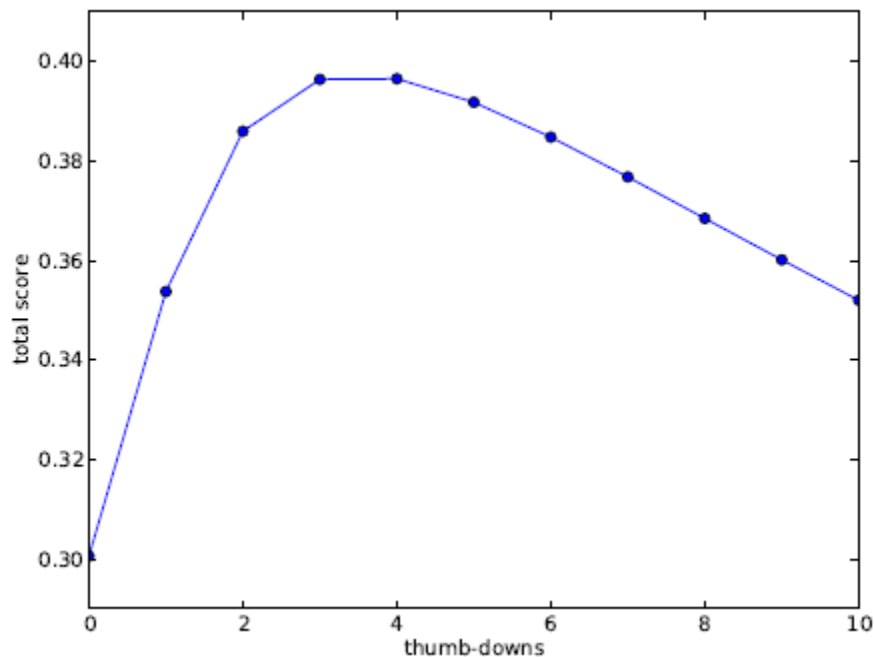
# Axiomatic Examination

- Difference
  - Axiom 1; ~~Axiom 2~~.
- Proportion
  - ~~Axiom 1~~; ~~Axiom 2~~.
- Absolute Discounting
  - ~~Axiom 1~~; ~~Axiom 2~~.
- Jelinek-Mercer
  - ~~Axiom 1~~; ~~Axiom 2~~.

$$n_{\downarrow} = 0$$

# Axiomatic Examination

- **Proposition.** The Wilson Interval method violates both Axiom 1 and Axiom 2.



The Wilson interval  $s(n_{\uparrow}, n_{\downarrow})$  with  $n_{\uparrow} = 1$ .



# Axiomatic Examination

- **Theorem.** The Dirichlet Prior smoothing method satisfies both Axiom 1 and Axiom 2.
  - Corollary 1. The Laplace smoothing method ...
  - Corollary 2. The Lidstone smoothing method ...

# Axiomatic Examination

	<b>Increasing Total Utility</b>	<b>Diminishing Marginal Utility</b>
Difference	✓	×
Proportion	×	×
Wilson Interval	×	×
Absolute Discounting	×	×
Jelinek-Mercer	×	×
Dirichlet Prior	✓	✓

# Conclusions

- Contribution
  - An Information Retrieval Approach to User-Rating based Ranking of Items
    - Probability Ranking Principle
    - Statistical Language Modelling
  - An Axiomatic Examination of the Existing and Proposed Methods
    - Increasing Total Utility
    - Decreasing Marginal Utility

Dirichlet Prior smoothing

# Conclusions

- Generalisations
  - Graded Ratings
    - => Multiple Thumb-Ups and Thumb-Downs
      - In a 5-star system: \*\*\* = 3↑ + 2↓
    - Learning the Weights from Click-Through Data
  - Ageing of User-Ratings
    - Time-Sensitive Language Modelling (ICTIR-2009)

# Question Time

(?\_?)

Thank You

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