Automata and Formal Languages

Peter Wood

Department of Computer Science and Information Systems
Birkbeck, University of London
ptw@dcs.bbk.ac.uk

Outline

Motivation and Background

Automata

Grammars

Regular Expressions

Example of Research

Conclusion

Doing Research

- analysing problems/languages
- computability/solvability/decidability
  — is there an algorithm?
- computational complexity
  — is it practical?
- expressive power
  — are there things that cannot be expressed?
- formal languages provide well-studied models

Formal Languages

- given a finite *alphabet* (set) of symbols Σ
  — e.g., $\Sigma = \{0, 1\}$
- a *string* is a sequence (concatenation) of symbols
  — e.g., 0101
- all finite strings over Σ are denoted by Σ^*
  — e.g., $\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, \ldots\}$
- *language* L over Σ is just a subset of Σ^*
  — e.g., L_1: strings with an even number of 1's
  — e.g., L_0: strings representing valid Java programs (over an alphabet of all legal symbols in Java)
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- are there finite representations for infinite languages?
- yes, *grammars* (generative) and *automata* (recognition)

Automata

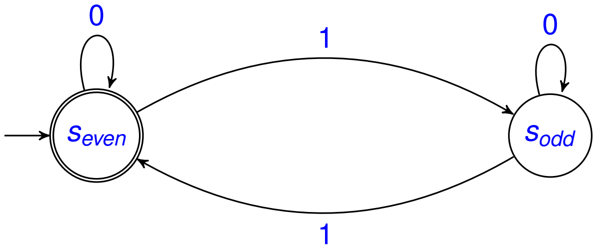
- device (machine) for recognising (accepting) a language
- provide models of computation
- automaton comprises *states* and *transitions* between states
- automaton is given a string as input
- automaton M *accepts* a string w by halting in an accept/final state, when given w as input
- language $L(M)$ *accepted* by automaton M is the set of all strings which M accepts

Types of Automata

- finite state automaton
  - deterministic
  - nondeterministic
- pushdown automaton
- linear-bounded automaton
- Turing machine

Example of a Finite State Automaton

- L_1 (strings with an even number of 1's) can be recognised by the following FSA
  - 2 states s_{even} and s_{odd}
  - 4 transitions
  - s_{even} is both the initial and final state



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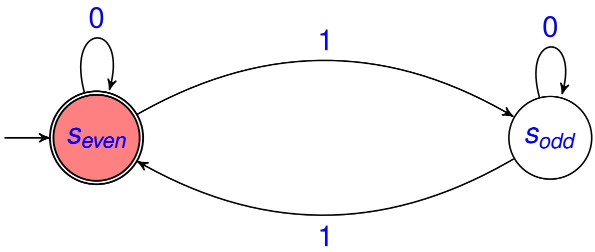

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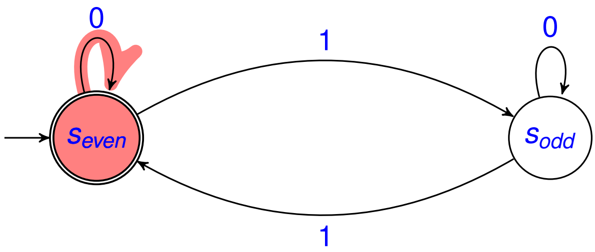

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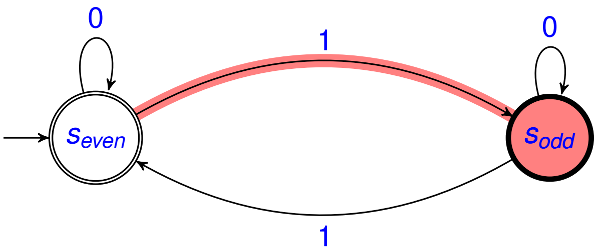

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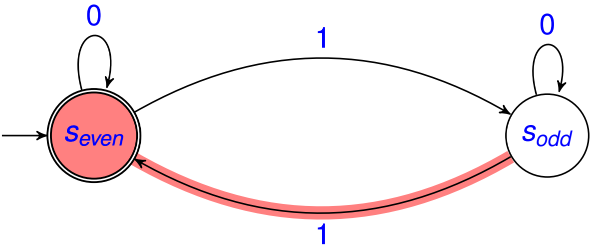

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Grammars

- *grammars* generate languages using:
  - symbols from alphabet Σ (called *terminals*)
  - set N of *nonterminals* (one designated as *starting*)
  - set P of *productions*, each of the form

$$U \to V$$

  where U and V are (loosely) strings over $\Sigma \cup N$
- a string (sequence of terminals) w is generated by G if there is a *derivation* of w using G, starting from the *starting* nonterminal of G
- language *generated* by grammar G, denoted $L(G)$, is the set of strings which can be derived using G

Grammar Example

- L_1 (strings with an even number of 1's) can be generated by a grammar with productions

$$\begin{aligned} S &\rightarrow \epsilon \\ S &\rightarrow 0S \\ S &\rightarrow 1T \\ T &\rightarrow 0T \\ T &\rightarrow 1S \end{aligned}$$

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Uses of Grammars

- to specify syntax of programming languages
- in natural language understanding
- in pattern recognition
- to specify schemas (types) for tree-structured data, e.g., XML, JSON
- in data compression
- ...

Hierarchy of Grammars and Languages

- restrictions on productions give different *types* of grammars
  - *regular* (type 3)
  - *context-free* (type 2)
  - *context-sensitive* (type 1)
  - *phrase-structure* (type 0)
- for context-free, e.g., left side must be single nonterminal
- no restrictions for phrase-structure
- language is of type i iff there is a grammar of type i which generates it

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- $L_3 = \{ww \mid w \in \{0, 1\}^*\}$ is context-sensitive, but not context-free
- there exists a phrase-structure (recursive) language which is not context-sensitive

Complexity of Grammar Problems

Problem	Type			
	3	2	1	0
Is $w \in L(G)$?	P	P	PSPACE	U
Is $L(G)$ empty?	P	P	U	U
Is $L(G_1) \equiv L(G_2)$?	PSPACE	U	U	U

- P: decidable in polynomial time
- PSPACE: decidable in polynomial space (and *complete* for PSPACE: at least as hard as NP-complete)
- U: undecidable
- so type of grammar has significant effect on complexity

Relationships between Languages and Automata

A language is

regular			finite-state
context-free	iff		pushdown
context-sensitive	accepted by		linear-bounded
phrase-structure			Turing machine

Regular Expressions

- algebraic notation for denoting regular languages
- use $\circ$ (concatenation), $\cup$ (union) and $*$ (closure) operators
- L_1 denoted by RE $0^* \cup (0^* \circ 1 \circ 0^* \circ 1 \circ 0^*)^*$
- given RE R, the set of strings it denotes is $L(R)$
- pattern matching in text
- query languages for XML or RDF

Using Regular Expressions to Query Graphs

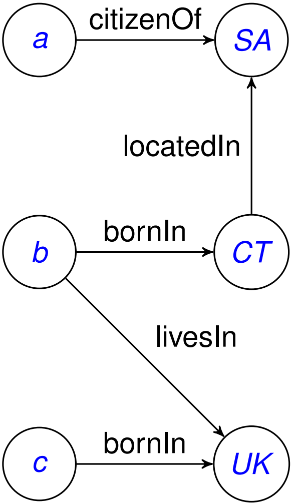
Graphs (networks) are widely used for representing data

- social networks
- transportation and other networks
- geographical information
- semistructured data (e.g., XML and JSON)
- (hyper)document structure
- semantic associations in criminal investigations
- bibliographic citation analysis
- pathways in biological processes
- knowledge representation (e.g. semantic web)
- program analysis
- workflow systems
- data provenance
- ...

Using Regular Expressions to Query Graphs

- (my PhD thesis!)
- usually regular expressions used for string search
- consider data represented by a directed graph of labelled nodes and labelled edges
- regular expressions can express *paths* we are interested in
- sequence of edge labels rather than sequence of symbols (characters)
- a query using regular expression R can ask for all nodes connected by a path whose concatenation of edge labels is in $L(R)$

Graph *G* (where nodes represent people and places):



Regular expression

$R = \text{citizenOf} \cup ((\text{bornIn} \cup \text{livesIn}) \circ \text{locatedIn}^*)$

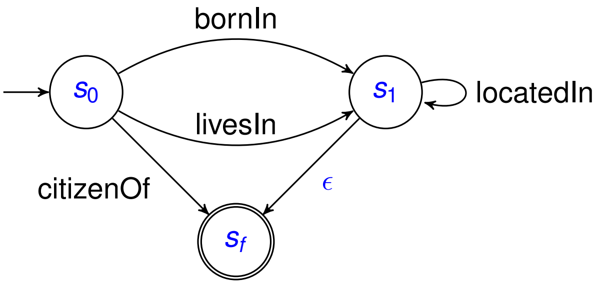
asks for paths of edges between a person x and a place y such that

- x is a citizenOf y, or
- x is bornIn or livesIn y, or
- x is bornIn or livesIn a place that is locatedIn y

Regular path query evaluation

- REGULAR PATH PROBLEM
  Given graph G, pair of nodes x and y and regular expression R, is there a path from x to y satisfying R?
- algorithm:
  - construct a nondeterministic finite automaton (NFA) M accepting $L(R)$
  - assume M has initial state s_0 and final state s_f
  - consider G as an NFA with initial state x and final state y
  - form the "intersection" (or "product") I of G and M
  - check if there is a path from (x, s_0) to (y, s_f)
- Each step can be done in PTIME, so REGULAR PATH PROBLEM has PTIME complexity

NFA M for R = citizenOf $\cup$ ((bornIn $\cup$ livesIn) $\circ$ locatedIn*)



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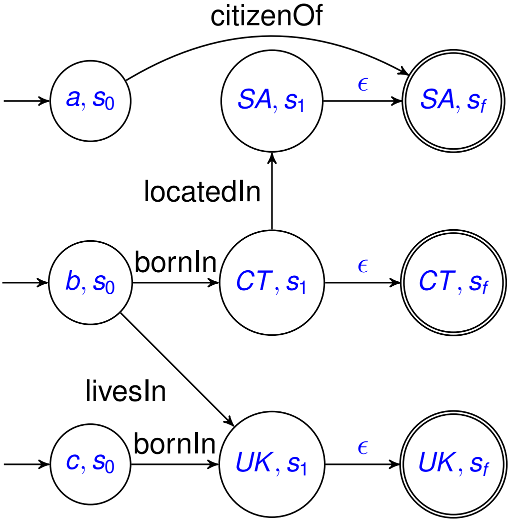

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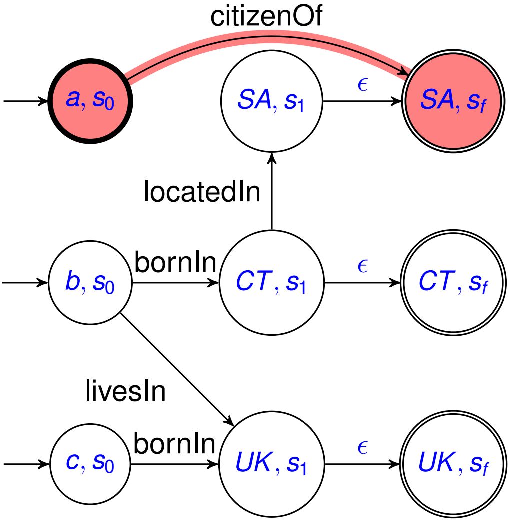

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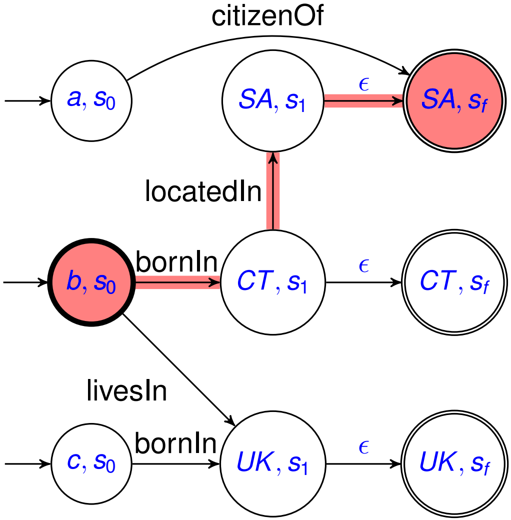

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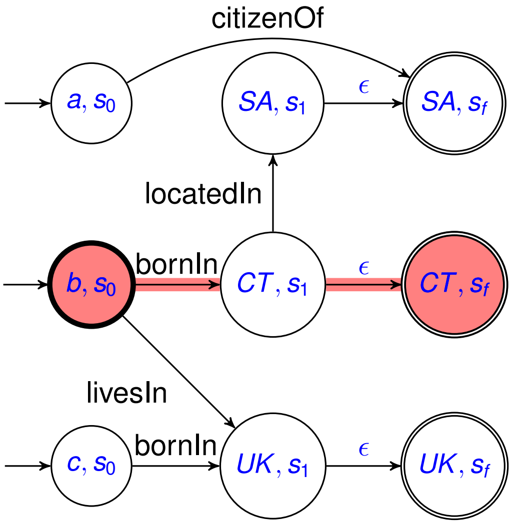

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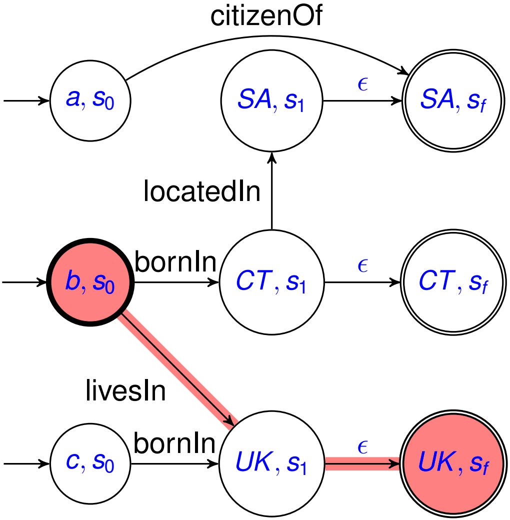

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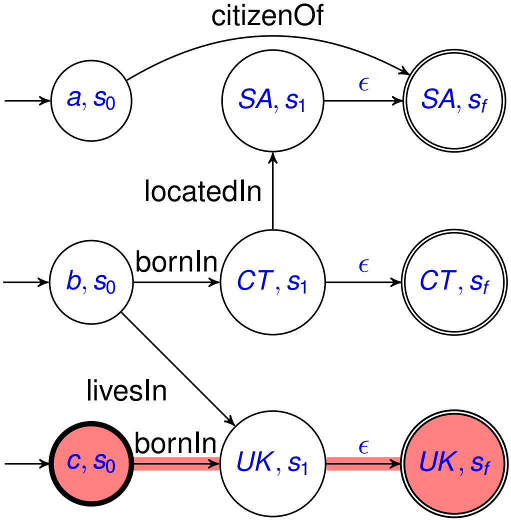

Regular simple path queries

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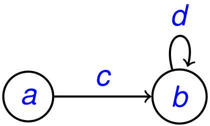
- REGULAR SIMPLE PATH PROBLEM is NP-complete [Mendelzon & Wood (1989)]
- there can be a path from x to y satisfying R but no simple path satisfying R, e.g., $R = (c \circ d)^*$



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- then might look at restricted forms of REs—we looked at those corresponding to languages closed under *abbreviations*
- then one might consider a combination of graphs and REs—we looked at graphs whose cycle structure does not *conflict* with the RE
- finally showed that conflict-freedom is a generalisation:
  - no RE conflicts with any DAG
  - an RE closed under abbreviations never conflicts with any graph

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  - other times this problem itself is NP-hard
- use heuristic approaches

Conclusion

- is my system/language more *powerful* than others?
- is my system/language more *efficient* than others?
- expressive power or computational complexity can be studied by relating them to
  - formal language theory: languages, grammars, automata, ...
- tradeoff between expressive power and computational complexity
- consider restrictions of difficult problems or giving up exact solutions

References

- Aho, Hopcroft and Ullman, "*The Design and Analysis of Computer Algorithms*," Addison-Wesley, 1974
- Garey and Johnson, "*Computers and Intractability: A Guide to the Theory of NP-Completeness*," W. H. Freeman and Company, 1979
- Lewis and Papadimitriou, "*Elements of the Theory of Computation*," Prentice-Hall, 1981
- Mendelzon and Wood, "Finding Regular Simple Paths in Graph Databases," *SIAM J. Computing*, Vol. 24. No. 6, 1995, pp. 1235–1258
- Sipser, "*Introduction to the Theory of Computation*," PWS Publishing Company, 1997