When are Description Logic Knowledge Bases Indistinguishable?

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joint work with **Elena Botoeva, Vladislav Ryzhikov, Frank Wolter** and **Michael Zakharyaschev**

Σ-Query Inseparability

Description Logic Knowledge Base $\mathcal{K}$

TBox $\mathcal{T}$	ABox $\mathcal{A}$
Minivan $\sqsubseteq$ Automobile Hybrid $\sqsubseteq$ Automobile Automobile $\sqsubseteq$ $\exists$poweredBy.Engine Hybrid $\sqsubseteq$ $\exists$poweredBy.EEngine $\sqcap$ $\exists$poweredBy.ICEngine EEngine $\sqsubseteq$ Engine ICEngine $\sqsubseteq$ Engine	Hybrid(toyota_highlander) Minivan(toyota_highlander) Minivan(nissan_note)

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EEngine $\sqsubseteq$ Engine
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ABox $\mathcal{A}$

Hybrid(toyota_highlander)
Minivan(toyota_highlander)
Minivan(nissan_note)

conjunctive query (CQ): $q(x) = \exists y\,(\text{Minivan}(x) \wedge \text{poweredBy}(x, y) \wedge \text{EEngine}(y))$

certain answers: $x \mapsto$ toyota_highlander

'$q(a)$ holds in all models of $\mathcal{K}$'

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a signature Σ is a set of concept and role names

$\mathcal{K}_1$ and $\mathcal{K}_2$ are **Σ-query inseparable** $(\mathcal{K}_1 \equiv_\Sigma \mathcal{K}_2)$ if
any CQ formulated in Σ has the same answers over $\mathcal{K}_1$ and $\mathcal{K}_2$

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   Automobile $\sqsubseteq$ Car   Engine $\sqsubseteq$ Motor
   Hybrid $\sqsubseteq$ HybridCar   EEngine $\sqsubseteq$ ElectricMotor   poweredBy $\sqsubseteq$ hasMotor

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4. $\mathcal{K}'$ is a **uniform interpolant** of $\mathcal{K}$ w.r.t. Σ if sig$(\mathcal{K}') \subseteq \Sigma$ and $\mathcal{K}' \equiv_\Sigma \mathcal{K}$
result of **forgetting** sig$(\mathcal{K}) \setminus \Sigma$

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KB Σ-query inseparability	TBox Σ-query inseparability	Σ-concept inseparability
$(\mathcal{T}_1, \mathcal{A}_1) \equiv_\Sigma (\mathcal{T}_2, \mathcal{A}_2)$	$(\mathcal{T}_1, \mathcal{A}) \equiv_\Sigma (\mathcal{T}_2, \mathcal{A})$ **for all $\mathcal{A}$**	indistinguishable by concept inclusions
indistinguishable by CQs		

Σ-Query Entailment in Horn DLs

$\mathcal{K}_1$ **Σ-query entails** $\mathcal{K}_2$ if

$$\mathcal{K}_2 \models q(\vec{a}) \implies \mathcal{K}_1 \models q(\vec{a}), \quad \text{for any } \Sigma\text{-CQ } q \text{ and any } \vec{a} \text{ in ind}(\mathcal{K}_2)$$

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$$\mathcal{T}_1 = \{ A \sqsubseteq \exists S, \quad \exists S^- \sqsubseteq \exists T.\exists S, \quad \exists Q^- \sqsubseteq \exists R, \quad S \sqsubseteq Q, \quad T \sqsubseteq Q \}$$

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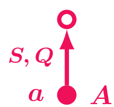

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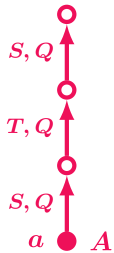

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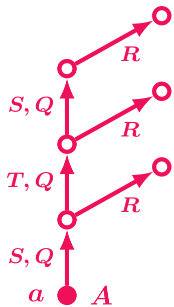

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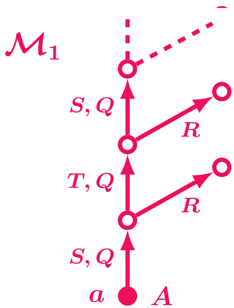

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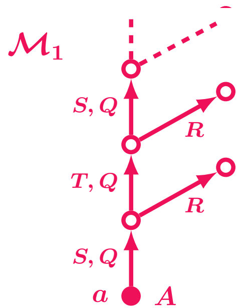

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for each $n > 0$, there is a Σ-homomorphism $h\colon \mathcal{M}_2^n \to \mathcal{M}_1$ (n-prefix of $\mathcal{M}_2$)

1. $h(a^{\mathcal{M}_2}) = a^{\mathcal{M}_1}$, for all $a \in \text{ind}(\mathcal{K}_2)$ that belong to a Σ-concept or a Σ-role
2. $t_\Sigma^{\mathcal{M}_2}(u) \subseteq t_\Sigma^{\mathcal{M}_1}(h(u))$, for all u
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(2 and 3:) preserves Σ-concepts & Σ-roles

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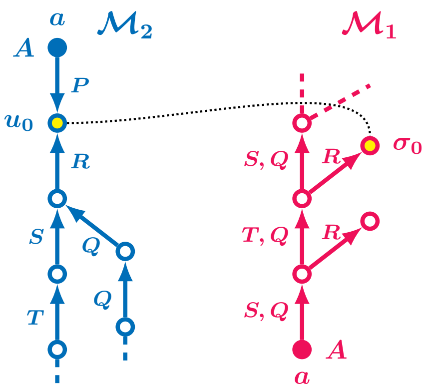

$u_0 \mapsto \sigma_0$ initial state 1 2

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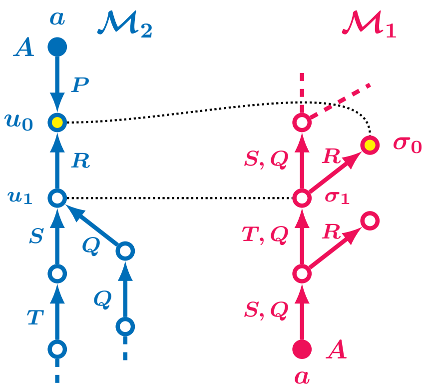

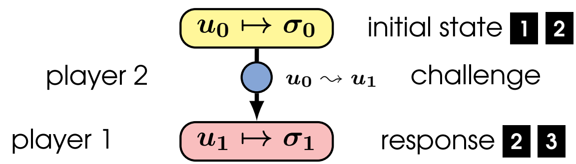

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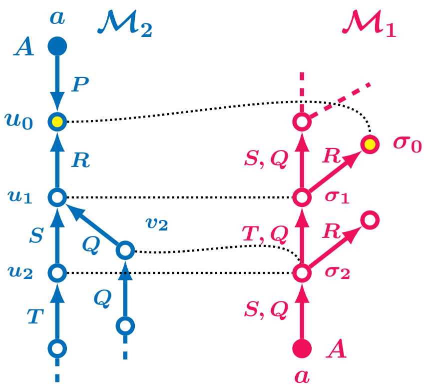

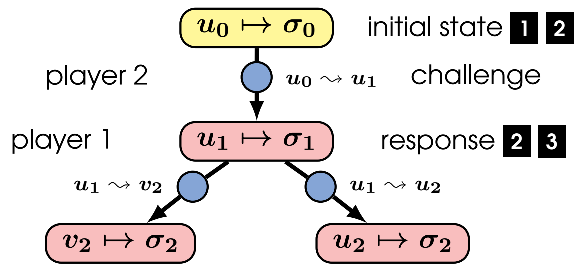

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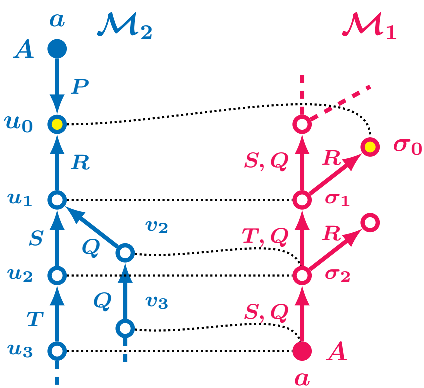

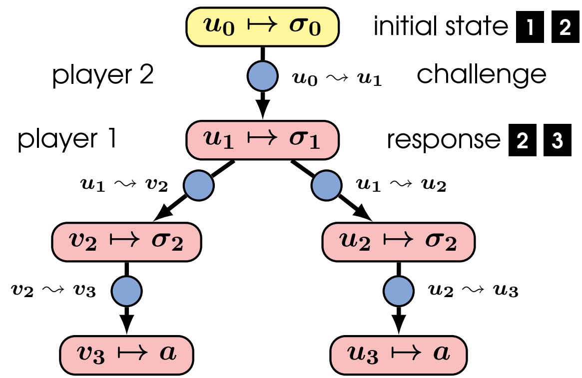

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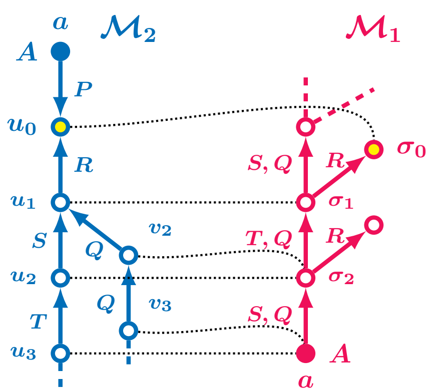

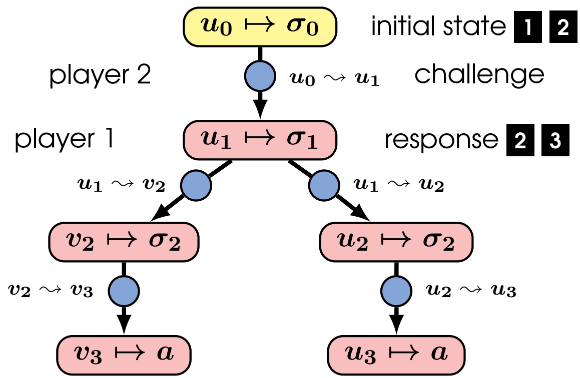

a. ABox individuals respect 1
b. for any $n < \omega$ and $u_0 \in \Delta^{\mathcal{M}_2}$, there is $\sigma_0^n \in \Delta^{\mathcal{M}_1}$ such that player 1 has a n-winning strategy starting from $(u_0 \mapsto \sigma_0^n)$

Games on Generating Structures

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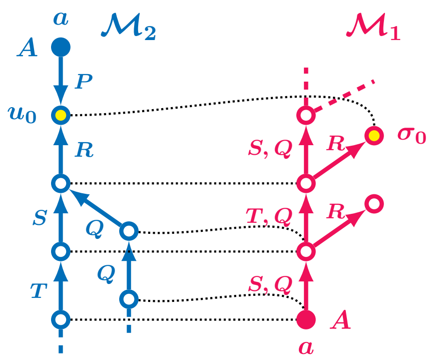

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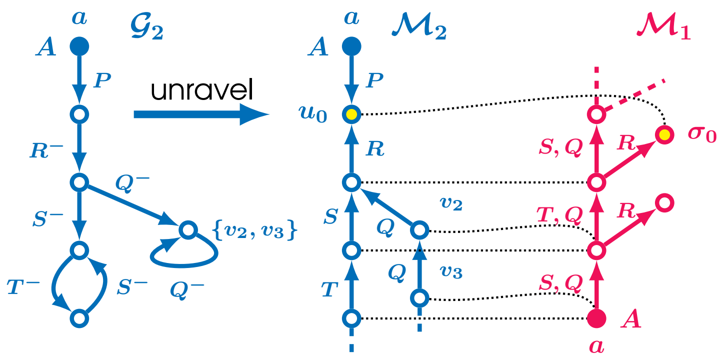

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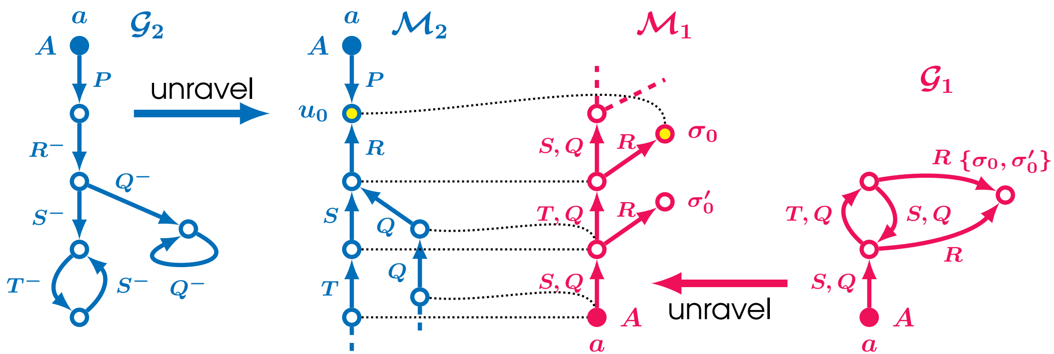

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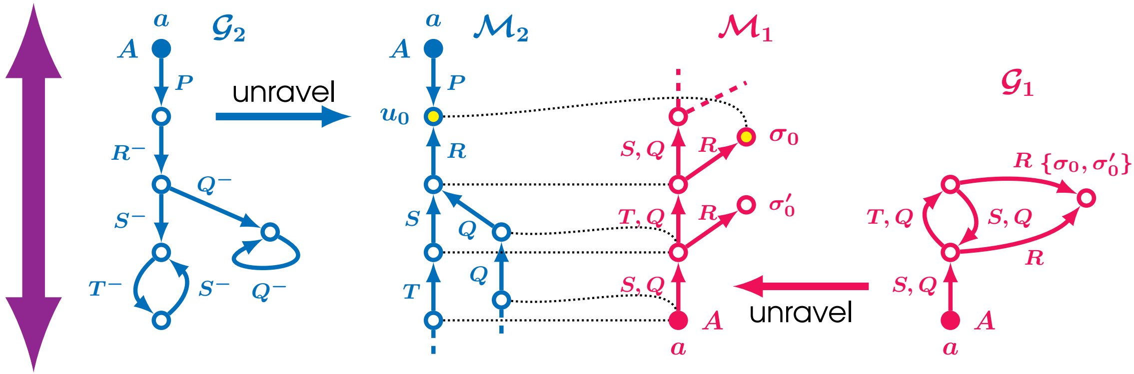

g for any $u_0 \in \Delta^{\mathcal{G}_2}$, there is $v_0 \in \Delta^{\mathcal{G}_1}$ such that player 1 has an ω-winning strategy starting from $(u_0 \mapsto v_0)$ in the **complex game on $\mathcal{G}_2$ and $\mathcal{G}_1$** (a combination of the backward and start-bounded games)

Games on Generating Structures

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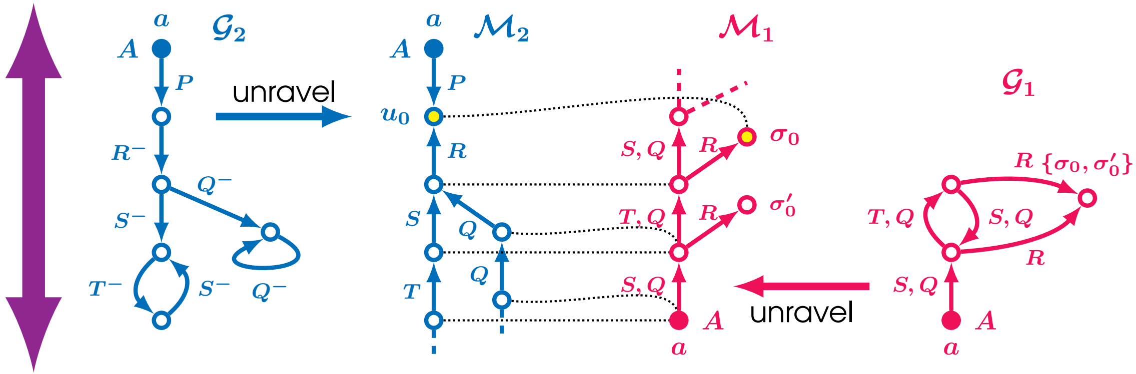

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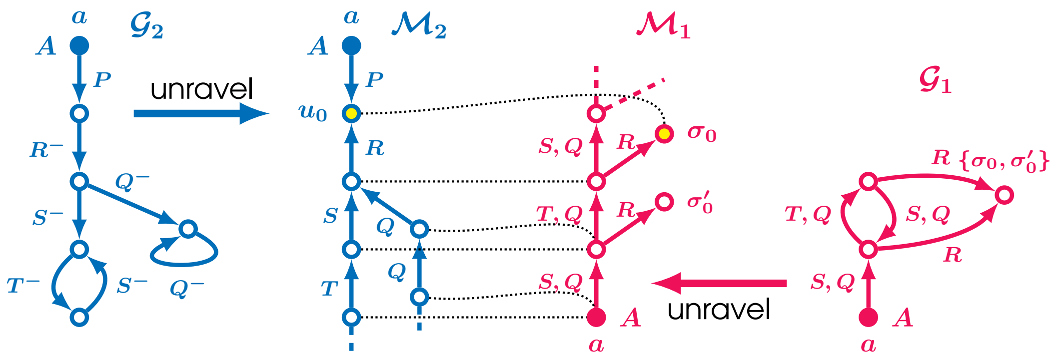

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f if **no role inverses** then, for any $u_0 \in \Delta^{\mathcal{G}_2}$, there is $v_0 \in \Delta^{\mathcal{G}_1}$ such that
player 1 has an ω-winning strategy starting from $(u_0 \mapsto v_0)$
polynomial time in the **forward game on $\mathcal{G}_2$ and $\mathcal{G}_1$**

Complexity Landscape

the size of the generating structure for $(\mathcal{T}, \mathcal{A})$:

- $|\mathcal{A}| \cdot 2^{p(|\mathcal{T}|)}$ in Horn-$\mathcal{ALCHI}$
- $|\mathcal{A}| \cdot p(|\mathcal{T}|)$ in $\mathcal{EL}$ and *DL-Lite* families

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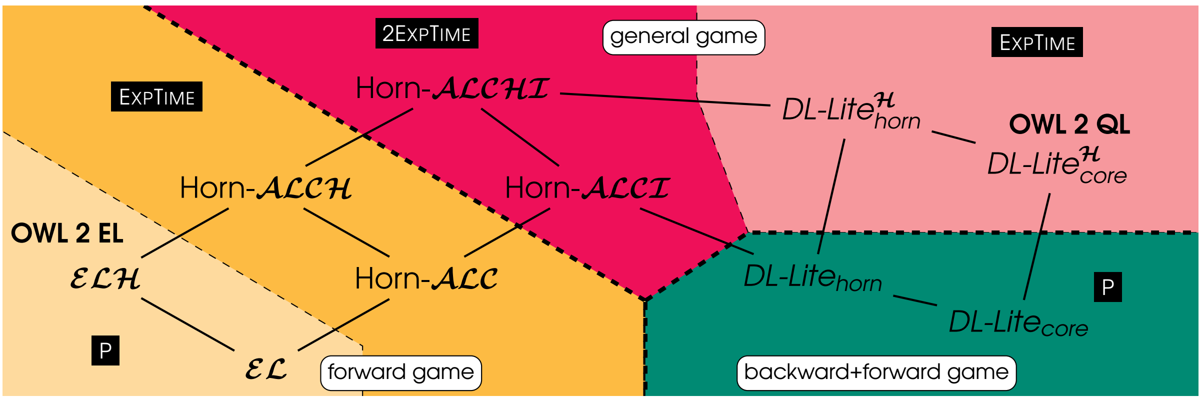

Applications of the Results

- Σ-query entailment for any of our DLs
  $\approx$ membership problem for **universal CQ-solutions** in the DL
  $\Longrightarrow$ the same complexity bounds
- complexity of query entailment between **OBDA specifications**
  $\approx$ complexity of Σ-query entailment **(Bienvenu & Rosati, 2015)**

Future Work

- approximations using forward games
- KB inseparability for more expressive DLs: Horn-$\mathcal{SHIQ}$ and $\mathcal{ALC}$