The Design of Teacher Assistance Tools in an Exploratory Learning Environment for Mathematics Generalisation

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Abstract. The MiGen project is designing and developing an intelligent, exploratory environment to support 11–14-year-old students in their learning of algebraic generalisation. Deployed within the classroom, the system is also providing tools to assist teachers in monitoring students' activities and progress. This paper describes the architectural design of these Teacher Assistance tools and gives a detailed description of one such tool, focussing in particular on the technologies and approaches chosen to implement the necessary functionality given the context of the project.
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1 Introduction

The MiGen project is co-designing, developing and evaluating with teachers and teacher educators a pedagogical and technical environment for improving 11–14-year-old students' learning of algebraic generalisation.[1] The idea of 'seeing the general through the particular' is a powerful way of introducing students to generalisation [1]. In MiGen, we are adopting a constructionist approach [2], allowing students to create and manipulate patterns and algebraic expressions and to perceive the relationships between them. Following [3], MiGen is based on the premise that learners build knowledge structures particularly well in situations where they are engaged in constructing public entities — in our context, these are patterns and expressions that they will share with other students. The MiGen system acts as a mediator of social interaction between students, and also between the teacher and the students, through which shared mathematical expressions are constructed and observed. The system provides not only a means by which students construct their own problem space but also a communicative function through which mathematical interpretations are made explicit and can be discussed with others.

There has been little previous work on supporting students in a constructionist context (e.g. [4–6]) while, conversely, it has been argued that considerable guidance is required to ensure learning in such contexts [7]. The exploratory nature of our learning environment requires that personalised feedback be provided to students by the system during their construction process. Also, since students are undertaking loosely-defined

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[1] See http://www.migen.org/ for details of the project's aims and background.

rather than structured tasks, teachers need to be assisted in monitoring their students' actions and progress by appropriate visualisation and notification tools [8]. The aim of these tools is to assist teachers in developing teaching strategies, informing them of students' progress, the appearance of common misconceptions, students who may be disengaged from the task and students who may be in difficulty. This will allow teachers to support learners in a personalised way, informing the teacher's own interventions in encouraging students to reflect on their constructions, on the feedback the system is giving them, on working towards specific goals, and in communicating and sharing their constuctions with others.

In [9], we described the conceptual model and architecture of the MiGen system as a whole. Here, we focus specifically on the design of the teacher assistance tools, motivating and describing their architectural design and giving a detailed description of one such tool. The paper is structured as follows. Section 2 gives a brief overview of the MiGen system and the overall system architecture to the level of detail necessary for this paper. Section 3 describes the architectural design of the teacher assistance tools, and gives a detailed description of the Student Tracking tool in particular. In Section 4 we discuss how this will be extended in the coming months with additional tools and the evaluation activities that we are planning.

To our knowledge, this work is the first targeted at visualising students' progress through constructionist learning tasks and at notifying teachers of students' attainment of specific landmarks as they are undertaking their constructions. We discuss the novelty and contribution of our work further in Section 4.

2   The MiGen System Context and Architecture

The MiGen system will be deployed within the classroom. During a lesson, students work on mathematics generalisation problems as selected by their teacher and presented to them by the system. While this is happening, the teacher may wish to view real-time representations of the students' activities and progress. At other times, teachers may also wish to access historical information about their students' progress as maintained by the system.

MiGen comprises a number of tools, which we describe here to the level of detail necessary for this paper. We refer the reader to the cited references for more details of some of these tools.

At the heart of the system is the **eXpresser**, a mathematical microworld [10, 11] which supports students in undertaking mathematics generalisation tasks. As part of a possibly larger activity sequence (see the Activity Tool below), students are asked to construct 'generalised patterns' using the eXpresser. Figure 1a and Figure 1b show two instances of an example pattern. The eXpresser supports students not only in their construction of the pattern but also in deriving expressions (rules) underpinning the pattern, e.g. the number of green tiles required (light grey) given the number of red tiles (dark grey).

The eXpresser microworld gives a lot of freedom to students, who may construct their patterns in a variety of ways for the same task. For example, if a student has constructed their pattern using a series of 'C' shapes as illustrated in Figure 1c, they

may derive an expression such as $5r + 3$ for the number of green tiles, where r is the number of red tiles. A range of other possible constructions that students may follow are shown in the remaining diagrams of Figure 1 which also show that the form of the resulting expressions (all of which are equivalent to $5r + 3$ of course) can vary widely. If r is a variable, all of these constructions are *general* and changing the current value of r will lead to the pattern changing appropriately too.

Figure 2 shows the eXpresser user interface. Students use building blocks that they construct from square unit tiles in order to make their patterns, which they can subsequently colour. During their construction, they make use of numbers which they can subsequently 'unlock' to turn them into variables, thus generalising their pattern. Both locked and unlocked numbers can be used in expressions. We refer the reader to [12] for further details of the eXpresser.

Microworlds are designed to provide opportunities for learners to develop complex cognitive skills rather than just knowledge of concepts in the subject domain [10, 11]. The tasks that students are usually asked to undertake are open-ended in nature, have multiple possible solutions, and encourage students to explore the construction environment and follow a variety of construction strategies. [13] describes MiGen's multi-layer learner model which, as well as modelling the attainment of concepts in the subject domain, also includes a 'layer' of knowledge comprising microworld-specific concepts that operationalise the concepts of the subject domain, as well as encompassing the affordances of the microworld itself.

In MiGen, tasks are designed to contextualise students' interaction with the eXpresser, including specific learning objectives that the learner should achieve as they undertake a task e.g. 'find a general expression to colour your pattern' — see Figure 2(G).

As a student interacts with the eXpresser, so a series of *landmarks* are automatically inferred by the system, which are then notified to the teacher via the Teacher Assistance tools (see below). There are two categories of landmark:

Task-independent (TI) landmarks occur when the system detects that specific actions or sequences of actions have been undertaken by a student e.g. 'student has placed a tile on the canvas', 'student has made a building block', 'student has unlocked a number'. The detection of TI landmarks is not dependent on knowledge specific to the task that the student is working on.

In contrast, *task-dependent* (TD) landmarks require intelligent reasoning to be applied to combinations of actions (this is done by the eGeneraliser — see below). TD landmarks require access to knowledge about the task and their detection may have a degree of uncertainty associated with it. Examples of TI landmarks are: 'student has made a plausible building block' (requires knowledge of the set of possible solutions to a task), 'student has unlocked too many numbers' (requires knowledge about how many variables a task needs), 'student has coloured their pattern generally' (requires reasoning on the student's expression).

The **Activity Tool** presents students with activity sequences targeting mathematics generalisation, as designed by the designer or teacher. These activity sequences include phases such as introduction to an exploratory learning task, undertaking a task using the
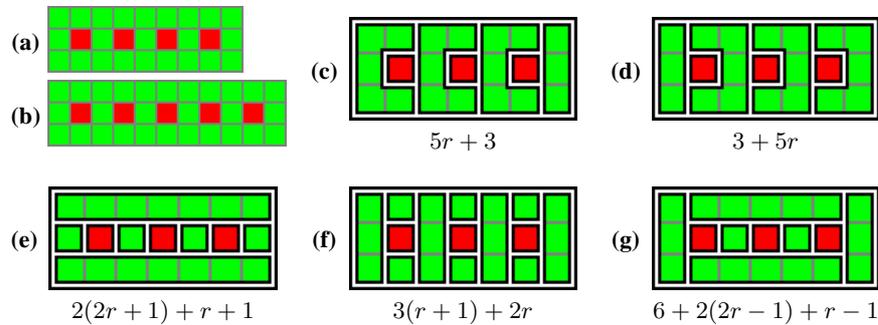
Fig. 1. **(a)–(b)** Instances of an example pattern and **(c)–(g)** several possible general constructions where each expression specifies the number of green tiles in terms of r, the number of red tiles.
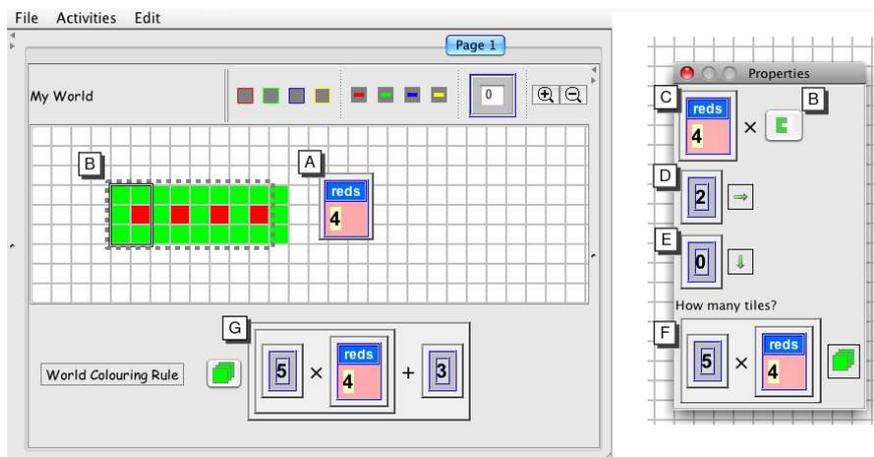

Fig. 2. Constructing a pattern in the eXpresser and describing it with a rule. Letters highlight the main features: (A) An 'unlocked' number that acts like a variable is given the name 'reds' and signifies the number of red (dark grey) tiles in the pattern. (B) Building block to be repeated to make a pattern. (C) Number of repetitions (in this case, the value of the variable 'reds'). (D,E) Number of grid squares to translate B to the right and down after each repetition. (F) Units of colour required to paint the pattern. (G) General expression that gives the total number of units of colour required to paint the whole pattern.

eXpresser, reflecting on a task, and sharing and discussing their constructions and rules with other students.

A **Task Design Tool** is planned for the near future. This will allow the designer or teacher to describe new maths generalisation tasks, and to input into the system a set of possible solutions for them, i.e. possible constructions and associated expressions. Currently, the set of possible solutions for a task is input into the system by the research team using the eXpresser.

Part of the description of a task is the set of learning objectives supported by the task, as selected by the designer/teacher from the total set of learning objectives supported by the system. Currently, such information describing tasks is entered directly into the MiGen database by the research team.

The total set of learning objectives supported by the system, and also the total set of landmarks that are detectable by the system, are easily extensible with new ones as the system is being developed and extended over time.

The **eGeneraliser** is a suite of intelligent components which take as their input information from the eXpresser as students are undertaking tasks, as well as information in the MiGen database relating to students (their learner model) and to tasks (the task description and the set of possible solutions). These intelligent components infer the occurrence of TD landmarks, generate real-time feedback for students (e.g. prompts to help students engage with a task, improve their solutions, generalise their solutions), and also update students' learner models during, and at the end of, each student's usage of the eXpresser tool.

A hybrid of case-based, pattern matching and rule-based reasoning is used in the eGeneraliser in order to infer the occurrence of TD landmarks and to update students' learner models [14]. For example, case-based reasoning applied to the set of known solutions and the student's own solution is used to determine if appropriate building blocks are being constructed and used correctly, while rule-based reasoning is used to determine if the student has coloured their pattern in a general way. We refer the reader to [15, 14] for further details of the eGeneraliser components and the inference mechanisms that they employ.

Finally, the **Teacher Assistance Tools** is a suite of tools aiming to assist the teacher in monitoring students' activities and progress and in intervening with additional support for students as she decides appropriate. An extensive requirements analysis for these tools has been undertaken over the past year with the teachers involved in the MiGen project, and this is driving the specification and co-design of these tools. We discuss these tools in greater detail in Section 3 below.

The overall MiGen system has the client-server architecture illustrated in Figure 3. The 'ball-and-socket' notation indicates a component which provides an API (the ball) and the components that use this — the socket(s). For example, the eGeneraliser Information Layer provides an API which is used by the eGeneraliser User Interface. The client software is executed on each student's and teacher's computer, while the server software is executed on one server computer. The architecture is based on Representational State Transfer (REST) [16], for the reasons discussed in detail in [9], including

simplicity of installation and operation of the system by schools, scalability, and ease of performance tuning. In particular we use Restlet, a lightweight Java REST framework.[2]
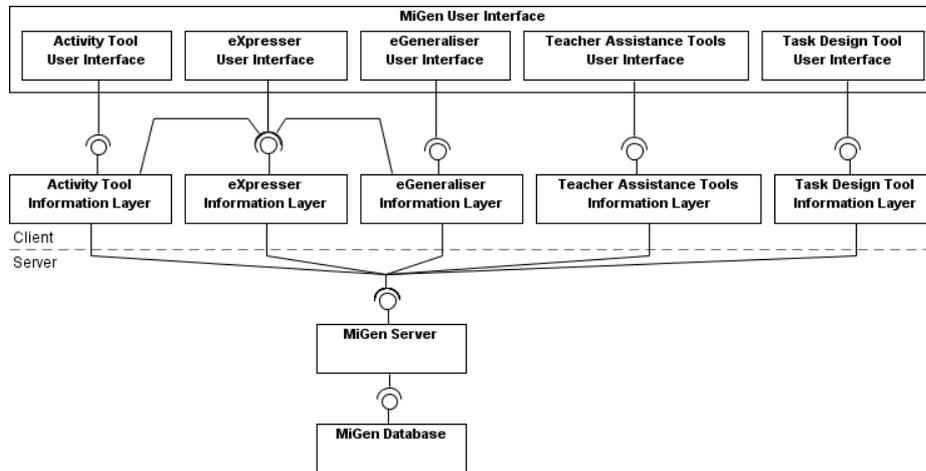

Fig. 3. MiGen system architecture.

As students interact with the Activity Tool and with the eXpresser, these tools' User Interface components update the data structures managed by their corresponding Information Layer components. These components in turn post information about students' actions to the MiGen Server, including the occurrence of TI landmarks. The MiGen Server stores a timestamp with each instance of a TI landmark.

The eGeneraliser Information Layer 'listens' to updates in the eXpresser Information Layer and from time to time infers the occurrence of a new TD landmark for this particular student, which it posts to the MiGen Server. It may also generate updates to the student's learner model which it similarly posts to the MiGen Server. It may also generate feedback messages appropriate for the student, which it displays to the student through the eGeneraliser UI and also posts to the MiGen Server. The MiGen Server stores a timestamp with each instance of a TD landmark, learner model update and feedback message.

The Teacher Assistance tools subscribe to the MiGen Server to be notified of the occurrence of TI and TD landmarks; more generally, they may subscribe to be notified of the occurrence of other updates as well e.g. to students' learner models. At present, the Teacher Assistance tools do not generate any information updates of their own and so do no posting to the MiGen Server — this may change in future versions of the system.

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[2] See `http://www.restlet.org/`.

3 The MiGen Teacher Assistance Tools

The Teacher Assistance tools derive their information from the MiGen database, via the MiGen Server. This information includes:

– log data relating to students' activities and constructions, as posted to the MiGen Server by the Activity tool and the eXpresser Information Layers;
– TI landmarks detected by the Activity tool and the eXpresser as students interact with these;
– TD landmarks inferred by the eGeneraliser;
– data posted to the MiGen Server by the eGeneraliser relating to updates that it has made to a student's learner model and feedback that it has generated for a student;
– the students' learner models; and
– the task descriptions and possible solutions.

Within any RESTful architecture such as the one implemented for MiGen, all data resources within the system are made available via their own unique URL and are addressable using this URL. For example, the occurrences of task landmarks are located at URLs of the form:

```
/UserSets/<id1>/ExpresserModels/<id2>/TaskLandmarkSet/TaskLandmarkOccurrences/
```

on the MiGen Server, where each `<id>` identifies a particular entity within the preceding collection of entities. For example, `/UserSets/` is the collection of all UserSets defined within the database and `/UserSets/4` is the UserSet within this collection with id '4'.

This URL demonstrates various entities defined within the MiGen database:

– There are a number of UserSets defined, with each UserSet representing a collection of MiGen users. Such a set can (and frequently does) consist of just a single user. However, in collaborative activities a pair of students (or possibly more) will be co-located at the same computer and the UserSet will consist of all of them. In this way, the intelligent support within the system is actually provided to whatever the population of each UserSet is. In principle, this could conceivably even be an entire class (including the teacher). It is a topic of future work to determine how learner modelling information obtained from multi-person UserSets can be used to populate the learner models of the individual students that comprise that set.
– Each UserSet can create an arbitrary number of ExpresserModels, with one ExpresserModel being created by each invocation of the eXpresser.
– Each ExpresserModel is associated with a TaskLandmarkSet which in turn comprises of a set of timestamped occurrences of landmarks.

As task landmarks are generated, the relevant components of the system post these occurrences to URLs of the above form. The MiGen Server then updates the database appropriately. Any such update to the database can also lead to the notification of all components of the architecture that have previously registered their interest in this data resource. In particular, the Teacher Assistance tools register their interest in URLs of this form, as well as other data resources, and the architecture ensures that these tools are notified when the data resources change. This includes notification of additions, deletions and updates.
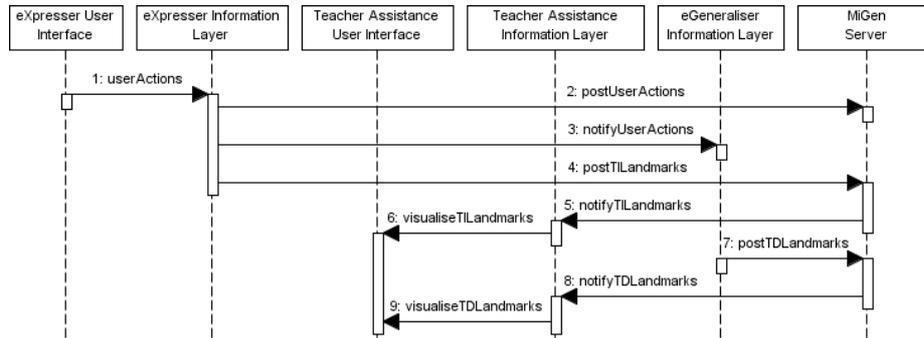
Fig. 4. UML sequence diagram demonstrating how the teacher assistance tools receive information about the occurrence of task-dependent (TD) and task-independent (TI) landmarks.

Figure 4 shows a UML Sequence Diagram illustrating this process involving both TI landmarks (from the eXpresser) and TD landmarks (detected by the eGeneraliser). As the User(Set) manipulates their constructions and expressions using the eXpresser User Interface, the underlying eXpresser Information Layer is updated (message ①). These events are posted to the MiGen Server ②. The eXpresser Information Layer also notifies the eGeneraliser Information Layer of the user's actions so that it can appropriately process them ③. The eXpresser Information Layer is also responsible for posting task-independent landmarks to the MiGen Server ④. The server notifies the Teacher Assistance Information Layer (⑤) which in turn updates its user interface ⑥. While task-independent landmarks occur directly and immediately from students' actions within the eXpresser, task-dependent landmarks are detected after the appropriate reasoning is applied within the eGeneraliser Information Layer, based on the actions of the user within the eXpresser so far. At various times therefore, the eGeneraliser Information Layer posts task-dependent landmarks to the MiGen Server (message ⑦). Since the Teacher Assistance Information Layer has registered interest (in this example) in receiving all landmark information, it is therefore notified ⑧ and the new landmark information is then used to update the visualisations within the Teacher Assistance User Interface ⑨.

Although this example pertains to the communication of task landmarks from the eXpresser and eGeneraliser to the Teacher Assistance tools, this pattern of messages also extends straightforwardly to landmark occurrences that are also posted by the Activity Tool to the server and that Teacher Assistance tools may also register to be notified of. This communication pattern therefore provides a general architectural template for supporting all of the information requirements of the Teacher Assistance tools in the MiGen system.

3.1 The Student Tracking Tool

So far, we have co-designed with teachers and implemented one of the envisaged suite of Teacher Assistance tools — the Student Tracking tool. This tool provides teachers

with information about individual students' attainment of TI and TD landmarks as they are undertaking a task using the eXpresser in the classroom, so that the teacher can follow students' progress on this task and intervene as she decides is appropriate in order to provide guidance and set new goals for individual students.
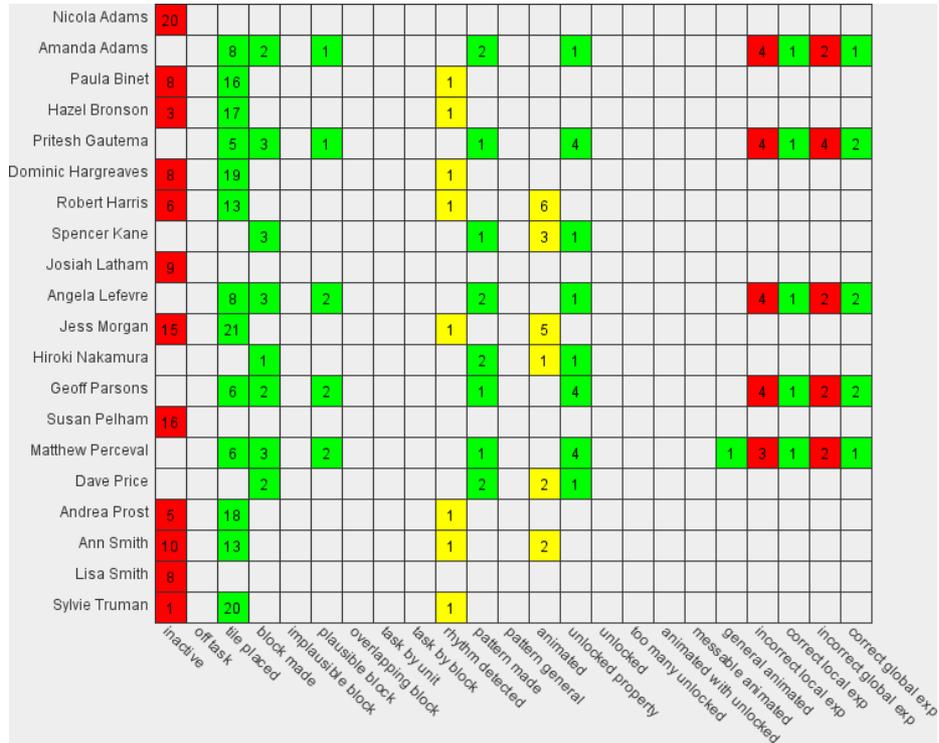
Student	inactive	off task	tile placed	block made	implausible block	plausible block	overlapping block	task by unit	task by block	rhythm detected	pattern made	pattern general	animated	unlocked property	unlocked	too many unlocked	animated with unlocked	messable animated	general animated	incorrect local exp	correct local exp	incorrect global exp	correct global exp
Nicola Adams	20																						
Amanda Adams			8	2		1					2		1							4	1	2	1
Paula Binet	8		16							1													
Hazel Bronson	3		17							1													
Pritesh Gautema			5	3		1					1		4							4	1	4	2
Dominic Hargreaves	8		19							1													
Robert Harris	6		13							1				6									
Spencer Kane				3							1			3	1								
Josiah Latham	9																						
Angela Lefevre			8	3		2					2		1							4	1	2	2
Jess Morgan	15		21							1				5									
Hiroki Nakamura				1							2			1	1								
Geoff Parsons			6	2		2					1		4							4	1	2	2
Susan Pelham	16																						
Matthew Perceval			6	3		2					1		4					1		3	1	2	1
Dave Price				2							2			2	1								
Andrea Prost	5		18							1													
Ann Smith	10		13							1				2									
Lisa Smith	8																						
Sylvie Truman	1		20							1													

Fig. 5. The Student Tracking User Interface showing the number of times each landmark has occurred for each student during the current task. Student names are listed in alphabetical order by last name. Landmarks are listed in order of expected first occurrence. See Table 1 for an explanation of the colours used. See Appendix A for an explanation of each landmark.

Figure 5 shows a screenshot of the Student Tracking tool. Each row shown represents the progress of the student named on the far left. Each column corresponds to a landmark as labelled at the bottom of the screenshot (we list the full set of TI and TD landmarks in Appendix A). A coloured cell indicates that a particular landmark has occurred for a particular student. The occurrence count is provided in the cell itself. The colouring of the cells indicates whether the landmark is regarded as being 'positive', 'negative' or 'neutral', as described in Table 1.

From Figure 5, it is immediately apparent that several users have been inactive at some point during their interaction with the system. However, many of these users have achieved other positive and/or neutral landmarks. For example, Ann Smith did have

Colour	Type	Description
Green (mid-grey)	Positive	The occurrence of such a landmark indicates that the actions of the student are in line with what would be expected from constructive interaction with the system.
Red (dark-grey)	Negative	The occurrence of such a landmark is regarded as an obstruction to constructive use of the system. For example, the student being off-task or inactive.
Yellow (light-grey)	Neutral	The occurrence of such a landmark indicates that the student is showing some understanding of how to use the system but it is not quite on the right (or wrong) track. For example, the student has constructed their pattern using repetitions of individual unit tiles, rather than a larger building block (this is the 'rhythm detected' landmark).

Table 1. Types of task landmark. The first column colours correspond to those used in Figure 5.

some periods of inactivity but she has also started placing tiles. She has also attempted to 'animate' her construction (this allows her to test out its generality by applying her expression to several instances of the pattern, as generated automatically by the system), even though she has not yet unlocked any numbers so as to in fact generalise her expression. All of these indicate to the teacher that Ann is having problems using the system, understanding the task or both. Lisa Smith, on the other hand, has just been inactive. Patterns of landmark occurrences such as these can assist the teacher in making decisions about which students need help the most.

In contrast to Ann and Lisa, we see that Angela Lefevre is progressing well with the task: she has placed tiles, made a plausible building block, built a pattern using this block and has unlocked one of its properties. She had problems specifying a correct local expression (this is an expression for the number of tiles of a particular colour in a specific pattern) and also with specifying the global expression correctly (this is the expression for the number of tiles of a particular colour in the construction overall). However, crucially, she has not yet animated her construction to test out its generality. The teacher can therefore use this information to remind her to try animating her construction. Angela would then see whether she has built it generally. This demonstrates therefore that not only can this representation show the teacher who needs help the most but can also provide more subtle information which can be used to ask students thought-provoking questions or further facilitate them on their way to successful completion of the task.

The data shown in Figure 5 is simulated user interaction data, semi-automatically generated by the authors in order to test the Student Tracking tool. Classroom-based activities are planned for the near future in order to evaluate the tool in context in collaboration with our teacher partners. For these trials, we are also planning to provide an additional visualisation within the Student Tracking tool which does not aggregate the total number of occurrences of each landmark but which instead presents the individual landmark occurrences for a student in the order in which they occur. This timeline-driven view will use the same data as the aggregated view shown above, but will tell a

more detailed 'story' about the way in which a student has interacted with the system in undertaking the task. We show this additional view in Figure 6. This view shows the sequential occurrence of each landmark for a group of students during their interaction with the task. Each landmark (coloured as before) is positioned in relation to the time at which the landmark occurred (with time increasing upwards). Taking Ann Smith as an example once again, this representation shows that, initially, Ann seemed to be disengaged from the task: she clicked on the animate button a couple of times despite her canvas being empty and then proceeded to place a handful of tiles. At this stage, the system detected that she was inactive for a while. If the teacher had viewed this representation at that point, she could have intervened and encouraged Ann to try placing single tiles to construct the pattern. She would then be able to see that Ann subsequently followed her advice and the system detected rhythm in Ann's placement of the tiles.

4 Discussion and Conclusions

In this paper we have described the architectural design of the Teacher Assistance tools of the MiGen system, an intelligent exploratory enviroment aiming to support 11–14-year-old students in their learning of algebraic generalisation. MiGen's Teacher Assistance tools aim to provide information to teachers about their students' activities and progress as they use the eXpresser and Activity tools, assisting them in the detection of students who may be experiencing difficulties, alerting them to students who seem to be demonstrating misconceptions and to those who may benefit from further, more challenging tasks. In general, the tools aim to allow the teacher to effectively facilitate the students' productive interaction with the system. The architectural design of the Teacher Assistance tools we have presented in this paper is generic and is applicable to the whole suite of such tools that are planned for development in the MiGen project in the coming months. We have given a detailed description of one such tool — the Student Tracking Tool. Several MiGen system components are used to infer the occurrence of task-independent and task-dependent landmarks of relevance to the teacher as students undertake tasks using the eXpresser microworld, and this information is presented in real-time to the teacher by the Student Tracking Tool.

To our knowledge, ours is the first work targeted at notifying the teacher about students' attainment of landmarks during constructionist learning tasks. There has of course been much work in developing tools that assist teachers' instructionist role, but to our knowledge there are no other tools that assist teachers in tracking their students' progress during constructionist learning. The work closest to ours is that of [17] which uses Web log data generated by course management systems to help instructors be aware of students' activities in distance learning classes; [18] which presents tools for helping teachers understand students' behaviour in adaptive tutorials through post-analysis of the system's data logs; and [19] which presents tools for teachers to visualise students' progress through simulation-based practical work sessions.

[17] uses techniques from information visualisation to represent multidimensional student tracking data, and our Student Tracking tool has a similar representation to their visualisations. However, they do not focus on detecting and visualising the occurrence of landmarks in open-ended constructionist learning tasks (which is our central con-

Fig. 6. A portion of the Landmark Timeline User Interface showing when landmarks occur for each user. This is a different view of the same data used in Figure 5. As before, the meaning of the colours is explained in Table 1.

cern in MiGen) but rather on visualising students' social and behavioural aspects, their progress with respect to the course schedule, and their performance on quizzes and assignments and the level of knowledge achieved for each domain concept of the course. We refer the reader to that paper for an extensive review of other work in visualisation of data collected by course management systems, some of which also uses data mining and intelligent techniques to analyse student data and generate feedback to users — though, again, none of this earlier work focusses on constructionist learning. [18] adopts a hybrid approach whereby part of the data mining effort is teacher-driven and part is automated, but like much other work in educational data mining it does not focus on monitoring students' ongoing progress through constructionist learning tasks. [19] composes a practical work session from a sequence of simulation-based problems (analogous to our eXpresser tasks undertaken within activity sequences); however, there are explicit conditions on simulation states that evaluate to correct/incorrect and there are explicit requests by learners for validation of conditions, neither of which is the case with our more open-ended constructionist learning tasks in MiGen.

A first version of the Student Tracking tool has been designed and implemented. The next phase of work on this tool will be its deployment and evaluation within full classroom-based activities. In particular, we wish to determine to what extent our expectations about how teachers will use the Student Tracking tool, as described in Section 3.1, are borne out in a practical classroom setting. How extensively will teachers consult the Student Tracking visualisation as their students are undertaking an eXpresser task? Will they find all the landmarks helpful or just a subset of them? Is the differentiation of landmarks into positive, neutral and negative helpful? How will teachers balance their use of the aggregated Student Tracking view and the more detailed timeline-based view? Will the tool allow teachers to easily spot students who may be disengaged from a task, students who may be in difficulty, the occurrence of common trends and misconceptions? We will gather data relating to these key questions by (a) observing our partner teachers in the classroom as they conduct lessons using MiGen, and (b) conducting post-lesson interviews with teachers.

We are also currently designing two further teacher assistance tools, the Student Progress Tool and the Classroom Dynamics tool.

The Student Progress Tool will provide teachers with longer-term information about learning objectives that their students have attained, as recorded within the students' learner models. [13] discusses in detail MiGen's multi-level learner model and the process by which this is updated as students undertake tasks. In brief, at the lowest level, the Task Short Term Model contains information about a student's construction strategies and attainment of particular Task Learning Objectives (LOs) during one task instance. The occurrence of task landmarks provides evidence for the attainment of Task LOs, as encapsulated within a set of declarative rules co-designed with teachers, which are applied by the eGeneraliser to the landmark occurrences and the current Task Short Term Model as a student is undertaking a task. The Task Long Term Model contains aggregated information about a student's strategies and achievement of Task LOs during multiple attempts at a particular task. The students' Microworld Long Term Model contains aggregated information about the students' attainment of LOs across all tasks undertaken. Finally, the student's Domain Long Term Model contains information about

the students' attainment of LOs as relating to the subject domain of mathematics generalisation. When a student finishes an instance of a task using the eXpresser, their Task Long Term Model, Microworld Long Term Model and Domain Long Term Model are updated by additional rule-based components within the eGeneraliser which successively infer updates to each higher 'layer' of the learner model based on the current values of this layer and the values of the layer below it. There are explicit correspondences between task LOs and domain LOs built into the system. For example, the task LO 'identify variants and invariants of constructions and expressions' corresponds to the domain LO 'identify variants and invariants' as extracted from the U.K. Maths National Curriculum. The Student Progress Tool will provide teachers with a visual overview of students' degree of attainment of task LOs and domain LOs over students' longer-term interaction with the eXpresser.

The Classroom Dynamics (CD) tool will provide a visual overview of students' locations in the classroom and their progress with respect to ongoing individual and collaborative activities. The CD tool will be driven partly by information posted by the Activity Tool relating to students' progress through activity sequences, partly from information posted by the eGeneraliser relating to students' construction strategies and attainment of learning objectives. The CD tool will support the teacher in selecting to view and monitor different aspects of this information and in using this to inform her choice of interventions, including choices of how best to group students together for collaborative activities on the basis of their individual work and progress.

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A Description of Landmarks

The table below provides a description of all the landmarks ordered as in Figure 5 and indicates whether they are task-independent (TI) or task-dependent (TD).

Landmark Name	TI/TD	Description
inactive	TI	The student has not been interacting with the system for some period of time.
off task	TD	The student is interacting with the system but is off-task.

Landmark Name	TI/TD	Description
block made	TI	The student has created a building block out of existing tiles/blocks/patterns.
tile placed	TI	The student has placed a single unit tile on the canvas.
implausible block	TD	The student has created an implausible building block which is unlikely to lead to a solution.
plausible block	TD	The student has created a plausible building block which could lead to a solution.
overlapping block	TD	The student has made a building block that will lead to overlaps when it is used.
task by unit	TD	The student has constructed the task pattern with unit tiles and some structure has been detected.
task by block	TD	The student has constructed the task pattern by creating a block and repeating it.
rhythm detected	TD	The student has placed unit tiles and some rhythm has been detected.
pattern made	TI	The student has made a pattern out of some of their existing tiles or building blocks.
pattern general	TD	The student has created a pattern and coloured it generally.
animated	TI	The student has clicked on the 'animate' button regardless of the content of their canvas.
unlocked property	TI	The student has unlocked a number within the property list of a constructed pattern.
unlocked	TI	The student has unlocked a number anywhere in their model.
too many unlocked	TD	The student has unlocked too many numbers for this particular task.
animated with unlocked	TI	The student has clicked on the 'animate' button and some numbers are currently unlocked.
messable animated	TD	The student has animated their construction but it is not general — it can be 'messed up'.
general animated	TD	The student has animated their construction and it is general.
incorrect local exp	TD	The student has created an incorrect expression for one of their patterns.
correct local exp	TD	The student has created a correct expression for one of their patterns.
incorrect global exp	TD	The student has created an incorrect expression for the construction overall.
correct global exp	TD	The student has created a correct expression for the construction overall.