

## Chapter 7

# EM Algorithms for Text Processing

Until the end of the 1980s, text processing systems tended to rely on large numbers of manually written rules to analyze, annotate, and transform text input, usually in a deterministic way. This rule-based approach can be appealing: a system’s behavior can generally be understood and predicted precisely, and, when errors surface, they can be corrected by writing new rules or refining old ones. However, rule-based systems suffer from a number of serious problems. They are brittle with respect to the natural variation found in language, and developing systems that can deal with inputs from diverse domains is very labor intensive. Furthermore, when these systems fail, they often do so catastrophically, unable to offer even a “best guess” as to what the desired analysis of the input might be.

In the last 20 years, the rule-based approach has largely been abandoned in favor of more data-driven methods, where the “rules” for processing the input are inferred automatically from large corpora of examples, called *training data*. The basic strategy of the data-driven approach is to start with a processing algorithm capable of capturing how any instance of the *kinds* of inputs (e.g., sentences or emails) can relate to any instance of the kinds of outputs that the final system should produce (e.g., the syntactic structure of the sentence or a classification of the email as spam). At this stage, the system can be thought of as having the potential to produce any output for any input, but they are not distinguished in any way. Next, a *learning algorithm* is applied which refines this process based on the training data—generally attempting to make the model perform as well as possible at predicting the examples in the training data. The learning process, which often involves iterative algorithms, typically consists of activities like ranking rules, instantiating the content of rule templates, or determining parameter settings for a given model. This is known as *machine learning*, an active area of research.

Data-driven approaches have turned out to have several benefits over rule-based approaches to system development. Since data-driven systems can be

trained using examples of the kind that they will eventually be used to process, they tend to deal with the complexities found in real data more robustly than rule-based systems do. Second, developing training data tends to be far less expensive than developing rules. For some applications, significant quantities of training data may even exist for independent reasons (e.g., translations of text into multiple languages are created by authors wishing to reach an audience speaking different languages, not because they are generating training data for a data-driven machine translation system). These advantages come at the cost of systems that often behave internally quite differently than a human-engineered system. As a result, correcting errors that the trained system makes can be quite challenging.

Data-driven information processing systems can be constructed using a variety of mathematical techniques, but in this chapter we focus on *statistical models*, which probabilistically relate inputs from an input set  $\mathcal{X}$  (e.g., sentences, documents, etc.), which are always *observable*, to annotations from a set  $\mathcal{Y}$ , which is the space of possible annotations or analyses that the system should predict. This model may take the form of either a *joint model*  $\Pr(x, y)$  which assigns a probability to every pair  $\langle x, y \rangle \in \mathcal{X} \times \mathcal{Y}$  or a *conditional model*  $\Pr(y|x)$ , which assigns a probability to every  $y \in \mathcal{Y}$ , given an  $x \in \mathcal{X}$ . For example, to create a statistical spam detection system, we might have  $\mathcal{Y} = \{\text{SPAM}, \text{NOTSPAM}\}$  and  $\mathcal{X}$  be the set of all possible email messages. For machine translation,  $\mathcal{X}$  might be the set of Arabic sentences and  $\mathcal{Y}$  the set of English sentences.<sup>1</sup>

There are three closely related, but distinct challenges in statistical text-processing. The first is model selection. This entails selecting a representation of a joint or conditional distribution over the desired  $\mathcal{X}$  and  $\mathcal{Y}$ . For a problem where  $\mathcal{X}$  and  $\mathcal{Y}$  are very small, one could imagine representing these probabilities in look-up tables. However, for something like email classification or machine translation, where the model space is infinite, the probabilities cannot be represented directly, and must be computed algorithmically. As an example of such models, we introduce *hidden Markov models* (HMMs), which define a joint distribution over sequences of inputs and sequences of annotations. The second challenge is *parameter estimation* or *learning*, which involves the application of an optimization algorithm and training criterion to select the parameters of the model to optimize the model's performance (with respect to the given training criterion) on the training data.<sup>2</sup> The parameters of a statistical model are the values used to compute the probability of some event described by the model. In this chapter we will focus on one particularly simple training criterion for

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<sup>1</sup>In this chapter, we will consider discrete models only. They tend to be sufficient for text processing, and their presentation is simpler than models with continuous densities. It should be kept in mind that the sets  $\mathcal{X}$  and  $\mathcal{Y}$  may still be countably infinite.

<sup>2</sup>We restrict our discussion in this chapter to models with finite numbers of parameters and where the learning process refers to setting those parameters. Inference in and learning of so-called *nonparameteric models*, which have an infinite number of parameters and have become important statistical models for text processing in recent years, is beyond the scope of this chapter.

parameter estimation, *maximum likelihood estimation*, which says to select the parameters that make the training data most probable under the model, and one learning algorithm that attempts to meet this criterion, called expectation maximization (EM). The final challenge for statistical modeling is the problem of *decoding*, or, given some  $x$ , using the model to select an annotation  $y$ . One very common strategy is to select  $y$  according to the following criterion:

$$y^* = \arg \max_{y \in \mathcal{Y}} \Pr(y|x) \quad (7.1)$$

In a conditional (or *direct*) model, this is a straightforward search for the best  $y$  under the model. In a joint model, the search is also straightforward, on account of the definition of conditional probability:

$$y^* = \arg \max_{y \in \mathcal{Y}} \Pr(y|x) = \arg \max_{y \in \mathcal{Y}} \frac{\Pr(x, y)}{\sum_{y'} \Pr(x, y')} = \arg \max_{y \in \mathcal{Y}} \Pr(x, y) \quad (7.2)$$

The specific form that the search takes will depend on how the model is represented. Our focus in this chapter will primarily be on the second problem: learning parameters for models, but we will touch on the third problem as well.

Machine learning is often categorized as either *supervised* or *unsupervised*. Supervised learning of statistical models simply means that the model parameters are estimated from training data consisting of pairs of inputs and annotations, that is  $\mathcal{Z} = \langle \langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots \rangle$  where  $\langle x_i, y_i \rangle \in \mathcal{X} \times \mathcal{Y}$  and  $y_i$  is the *gold standard* (i.e., correct) annotation of  $x_i$ . While supervised models often attain quite good performance, they are often uneconomical to use, since the training data requires each object that is to be classified (to pick a specific task),  $x_i$  to be paired with its correct label,  $y_i$ . In many cases, these gold standard training labels must be generated by a process of *expert annotation*, meaning that each  $x_i$  must be manually labeled by a trained individual. Even when the annotation task is quite simple for people to carry out (e.g., in the case of spam detection), the number of potential examples that could be classified (representing a subset of  $\mathcal{X}$ , which may of course be infinite in size) will far exceed the amount of data that can be annotated. As the annotation task becomes more complicated (e.g., when predicting more complex structures such as sequences of labels or when the annotation task requires specialized expertise), annotation becomes far more challenging.

Unsupervised learning, on the other hand, requires only that the training data consist of a representative collection of objects that should be annotated, that is  $\mathcal{Z} = \langle x_1, x_2, \dots \rangle$  where  $x_i \in \mathcal{X}$ , but *without* any example annotations. While it may at first seem counterintuitive that meaningful annotations can be learned without any examples of the desired annotations being given, the learning criteria and model structure (which crucially define the space of possible annotations  $\mathcal{Y}$  and the process by which annotations relate to observable inputs) make it possible to induce annotations by relying on regularities in the unclassified training instances. While a thorough discussion of unsupervised learning is beyond the scope of this book, we focus on a particular class of

algorithms—expectation maximization (EM) algorithms—that can be used to learn the parameters of a joint model  $\Pr(x, y)$  from incomplete data (i.e., data where some of the variables in the model cannot be observed; in the case of unsupervised learning, the  $y_i$ 's are unobserved). Expectation maximization algorithms fit naturally into the MapReduce paradigm, and are used to solve a number of problems of interest in text processing. Furthermore, these algorithms can be quite computationally expensive, since they generally require repeated evaluations of the training data. MapReduce therefore provides an opportunity not only to scale to larger amounts of data, but also to improve efficiency bottlenecks at scales where non-parallel solutions could be utilized.

This chapter is organized as follows. In Section 7.1, we describe maximum likelihood estimation for statistical models, show how this is generalized to models where not all variables are observable, and then introduce expectation maximization (EM). We describe hidden Markov models (HMMs) in Section 7.2, a very versatile class of models that uses EM for parameter estimation. Section 7.3 discusses how EM algorithms can be expressed in MapReduce, and then in Section 7.4 we look at a case study of word alignment for statistical machine translation. Section 7.5 examines similar algorithms that are appropriate for supervised learning tasks. This chapter concludes with a summary and pointers to additional readings.

## 7.1 Expectation Maximization

Expectation maximization (EM) algorithms [49] are a family of iterative optimization algorithms for learning probability distributions from incomplete data. They are extensively used in statistical natural language processing where one seeks to infer latent linguistic structure from unannotated text. To name just a few applications, EM algorithms have been used to find part-of-speech sequences, constituency and dependency trees, alignments between texts in different languages, alignments between acoustic signals and their transcriptions, as well as for numerous other clustering and structure discovery problems.

Expectation maximization generalizes the principle of maximum likelihood estimation to the case where the values of some variables are unobserved (specifically, those characterizing the latent structure that is sought).

### Maximum Likelihood Estimation

Maximum likelihood estimation (MLE) is a criterion for fitting the parameters  $\theta$  of a statistical model to some given data  $\mathbf{x}$ . Specifically, it says to select the parameter settings  $\theta^*$  such that the likelihood of observing the training data given the model is maximized:

$$\theta^* = \arg \max_{\theta} \Pr(\mathbf{X} = \mathbf{x}; \theta) \quad (7.3)$$

To illustrate, consider the simple marble game shown in Figure 7.1. In this game, a marble is released at the position indicated by the black dot, and it

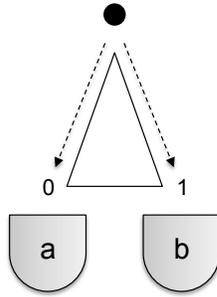


Figure 7.1: A simple marble game where a released marble takes one of two possible paths. This game can be modeled using a Bernoulli random variable with parameter  $p$ , which indicates the probability that the marble will go to the right when it hits the peg.

bounces down into one of the cups at the bottom of the board, being diverted to the left or right by the peg (indicated by a triangle) in the center. Our task is to construct a model that predicts which cup the ball will drop into. A “rule-based” approach might be to take exact measurements of the board and construct a physical model that we can use to predict the behavior of the ball. Given sophisticated enough measurements, this could certainly lead to a very accurate model. However, the construction of this model would be quite time consuming and difficult.

A statistical approach, on the other hand, might be to assume that the behavior of the marble in this game can be modeled using a Bernoulli random variable  $Y$  with parameter  $p$ . That is, the value of the random variable indicates whether path 0 or 1 is taken. We also define a random variable  $X$  whose value is the label of the cup that the marble ends up in; note that  $X$  is deterministically related to  $Y$ , so an observation of  $X$  is equivalent to an observation of  $Y$ .

To estimate the parameter  $p$  of the statistical model of our game, we need some *training data*, so we drop 10 marbles into the game which end up in cups  $\mathbf{x} = \langle b, b, b, a, b, b, b, b, b, a \rangle$ .

What is the maximum likelihood estimate of  $p$  given this data? By assuming that our samples are independent and identically distributed (i.i.d.), we can write the likelihood of our data as follows:<sup>3</sup>

$$\Pr(\mathbf{x}; p) = \prod_{j=1}^{10} p^{\delta(x_j, a)} (1 - p)^{\delta(x_j, b)} \quad (7.4)$$

$$= p^2 \cdot (1 - p)^8 \quad (7.5)$$

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<sup>3</sup>In this equation,  $\delta$  is the Kronecker delta function which evaluates to 1 where its arguments are equal and 0 otherwise.

Since  $\log$  is a monotonically increasing function, maximizing  $\log \Pr(\mathbf{x}; p)$  will give us the desired result. We can do this differentiating with respect to  $p$  and finding where the resulting expression equals 0:

$$\frac{d \log \Pr(\mathbf{x}; p)}{dp} = 0 \tag{7.6}$$

$$\frac{d[2 \cdot \log p + 8 \cdot \log(1 - p)]}{dp} = 0 \tag{7.7}$$

$$\frac{2}{p} - \frac{8}{1 - p} = 0 \tag{7.8}$$

Solving for  $p$  yields 0.2, which is the intuitive result. Furthermore, it is straightforward to show that in  $N$  trials where  $N_0$  marbles followed path 0 to cup  $a$ , and  $N_1$  marbles followed path 1 to cup  $b$ , the maximum likelihood estimate of  $p$  is  $N_1/(N_0 + N_1)$ .

While this model only makes use of an approximation of the true physical process at work when the marble interacts with the game board, it is an empirical question whether the model works well enough in practice to be useful. Additionally, while a Bernoulli trial is an extreme approximation of the physical process, if insufficient resources were invested in building a physical model, the approximation may perform better than the more complicated “rule-based” model. This sort of dynamic is found often in text processing problems: given enough data, astonishingly simple models can outperform complex knowledge-intensive models that attempt to simulate complicated processes.

## A Latent Variable Marble Game

To see where latent variables might come into play in modeling, consider a more complicated variant of our marble game shown in Figure 7.2. This version consists of three pegs that influence the marble’s path, and the marble may end up in one of three cups. Note that both paths 1 and 2 lead to cup  $b$ .

To construct a statistical model of this game, we again assume that the behavior of a marble interacting with a peg can be modeled with a Bernoulli random variable. Since there are three pegs, we have three random variables with parameters  $\theta = \langle p_0, p_1, p_2 \rangle$ , corresponding to the probabilities that the marble will go to the right at the top, left, and right pegs. We further define a random variable  $X$  taking on values from  $\{a, b, c\}$  indicating what cup the marble ends in, and  $Y$ , taking on values from  $\{0, 1, 2, 3\}$  indicating which path was taken. Note that the full joint distribution  $\Pr(X = x, Y = y)$  is determined by  $\theta$ .

How should the parameters  $\theta$  be estimated? If it were possible to observe the paths taken by marbles as they were dropped into the game, it would be trivial to estimate the parameters for our model using the maximum likelihood estimator—we would simply need to count the number of times the marble bounced left or right at each peg. If  $N_x$  counts the number of times a marble took path  $x$  in  $N$  trials, this is:

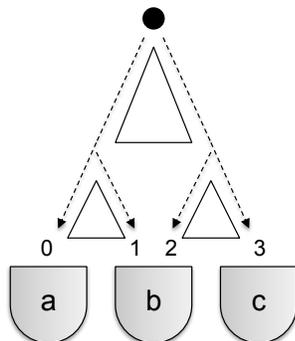


Figure 7.2: A more complicated marble game where the released marble takes one of four possible paths. We assume that we can only observe which cup the marble ends up in, not the specific path taken.

$$p_0 = \frac{N_2 + N_3}{N} \quad p_1 = \frac{N_1}{N_0 + N_1} \quad p_2 = \frac{N_3}{N_2 + N_3} \quad (7.9)$$

However, we wish to consider the case where the paths taken are *unobservable* (imagine an opaque sheet covering the center of the game board), but where we can see what cup a marble ends in. In other words, we want to consider the case where we have *partial* data. This is exactly the problem encountered in unsupervised learning: there is a statistical model describing the relationship between two sets of variables ( $X$ 's and  $Y$ 's), and there is data available from just one of them. Furthermore, such algorithms are quite useful in text processing, where latent variables may describe latent linguistic structures of the observed variables, such as parse trees or part-of-speech tags, or alignment structures relating sets of observed variables (see Section 7.4).

## MLE with Latent Variables

Formally, we consider the problem of estimating parameters for statistical models of the form  $\Pr(X, Y; \theta)$  which describe not only an observable variable  $X$  but a latent, or hidden, variable  $Y$ .

In these models, since only the values of the random variable  $X$  are observable, we define our optimization criterion to be the maximization of the *marginal* likelihood, that is, summing over all settings of the latent variable  $Y$ , which takes on values from set designated  $\mathcal{Y}$ :<sup>4</sup> Again, we assume that samples

<sup>4</sup>For this description, we assume that the variables in our model take on discrete values. Not only does this simplify exposition, but discrete models are widely used in text processing.

in the training data  $\mathbf{x}$  are i.i.d.:

$$\Pr(X = x) = \sum_{y \in \mathcal{Y}} \Pr(X = x, Y = y; \theta) \quad (7.10)$$

For a vector of training observations  $\mathbf{x} = \langle x_1, x_2, \dots, x_\ell \rangle$ , if we assume the samples are i.i.d.:

$$\Pr(\mathbf{x}; \theta) = \prod_{j=1}^{|\mathbf{x}|} \sum_{y \in \mathcal{Y}} \Pr(X = x_j, Y = y; \theta) \quad (7.11)$$

Thus, the maximum (marginal) likelihood estimate of the model parameters  $\theta^*$  given a vector of i.i.d. observations  $\mathbf{x}$  becomes:

$$\theta^* = \arg \max_{\theta} \prod_{j=1}^{|\mathbf{x}|} \sum_{y \in \mathcal{Y}} \Pr(X = x_j, Y = y; \theta) \quad (7.12)$$

Unfortunately, in many cases, this maximum cannot be computed analytically, but the iterative hill-climbing approach of expectation maximization can be used instead.

## Expectation Maximization

Expectation maximization (EM) is an iterative algorithm that finds a successive series of parameter estimates  $\theta^{(0)}, \theta^{(1)}, \dots$  that improve the marginal likelihood of the training data. That is, EM guarantees:

$$\prod_{j=1}^{|\mathbf{x}|} \sum_{y \in \mathcal{Y}} \Pr(X = x_j, Y = y; \theta^{(i+1)}) \geq \prod_{j=1}^{|\mathbf{x}|} \sum_{y \in \mathcal{Y}} \Pr(X = x_j, Y = y; \theta^{(i)}) \quad (7.13)$$

The algorithm starts with some initial set of parameters  $\theta^{(0)}$  and then updates them using two steps: expectation (E-step), which computes the posterior distribution over the latent variables given the observable data  $\mathbf{x}$  and a set of parameters  $\theta^{(i)}$ ,<sup>5</sup> and maximization (M-step), which computes new parameters  $\theta^{(i+1)}$  maximizing the expected log likelihood of the joint distribution with respect to the distribution computed in the E-step. The process then repeats with these new parameters. The algorithm terminates when the likelihood remains unchanged.<sup>6</sup> In more detail, the steps are as follows:

<sup>5</sup>The term ‘expectation’ is used since the values computed in terms of the posterior distribution  $\Pr(\mathbf{y}|\mathbf{x}; \theta^{(i)})$  that are required to solve the M-step have the form of an expectation (with respect to this distribution).

<sup>6</sup>The final solution is only guaranteed to be a *local maximum*, but if the model is fully convex, it will also be the global maximum.

**E-step.** Compute the posterior probability of each possible hidden variable assignments  $y \in \mathcal{Y}$  for each  $x \in \mathcal{X}$  and the current parameter settings, weighted by the relative frequency with which  $x$  occurs in  $\mathbf{x}$ . Call this  $q(X = x, Y = y; \theta^{(i)})$  and note that it defines a joint probability distribution over  $\mathcal{X} \times \mathcal{Y}$  in that  $\sum_{(x,y) \in \mathcal{X} \times \mathcal{Y}} q(x, y) = 1$ .

$$q(x, y; \theta^{(i)}) = f(x|\mathbf{x}) \cdot \Pr(Y = y|X = x; \theta^{(i)}) \quad (7.14)$$

$$= f(x|\mathbf{x}) \cdot \frac{\Pr(x, y; \theta^{(i)})}{\sum_{y'} \Pr(x, y'; \theta^{(i)})} \quad (7.15)$$

**M-step.** Compute new parameter settings that maximize the expected log of the probability of the joint distribution under the  $q$ -distribution that was computed in the E-step:

$$\theta^{(i+1)} = \arg \max_{\theta'} \mathbb{E}_{q(X=x, Y=y; \theta^{(i)})} \log \Pr(X = x, Y = y; \theta') \quad (7.16)$$

$$= \arg \max_{\theta'} \sum_{(x,y) \in \mathcal{X} \times \mathcal{Y}} q(X = x, Y = y; \theta^{(i)}) \cdot \log \Pr(X = x, Y = y; \theta') \quad (7.17)$$

We omit the proof that the model with parameters  $\theta^{(i+1)}$  will have equal or greater marginal likelihood on the training data than the model with parameters  $\theta^{(i)}$ , but this is provably true [78].

Before continuing, we note that the effective application of expectation maximization requires that both the E-step and the M-step consist of tractable computations. Specifically, summing over the space of hidden variable assignments must not be intractable. Depending on the independence assumptions made in the model, this may be achieved through techniques such as dynamic programming. However, some models may require intractable computations.

## An EM Example

Let's look at how to estimate the parameters from our latent variable marble game from Section 7.1 using EM. We assume training data  $\mathbf{x}$  consisting of  $N = |\mathbf{x}|$  observations of  $X$  with  $N_a$ ,  $N_b$ , and  $N_c$  indicating the number of marbles ending in cups  $a$ ,  $b$ , and  $c$ . We start with some parameters  $\theta^{(0)} = \langle p_0^{(0)}, p_1^{(0)}, p_2^{(0)} \rangle$  that have been randomly initialized to values between 0 and 1.

**E-step.** We need to compute the distribution  $q(X = x, Y = y; \theta^{(i)})$ , as defined above. We first note that the relative frequency  $f(x|\mathbf{x})$  is:

$$f(x|\mathbf{x}) = \frac{N_x}{N} \quad (7.18)$$

Next, we observe that  $\Pr(Y = 0|X = a) = 1$  and  $\Pr(Y = 3|X = c) = 1$  since cups  $a$  and  $c$  fully determine the value of the path variable  $Y$ . The posterior

probability of paths 1 and 2 are only non-zero when  $X$  is  $b$ :

$$\Pr(1|b; \theta^{(i)}) = \frac{(1 - p_0^{(i)})p_1^{(i)}}{(1 - p_0^{(i)})p_1^{(i)} + p_0^{(i)}(1 - p_2^{(i)})} \quad (7.19)$$

$$\Pr(2|b; \theta^{(i)}) = \frac{p_0^{(i)}(1 - p_2^{(i)})}{(1 - p_0^{(i)})p_1^{(i)} + p_0^{(i)}(1 - p_2^{(i)})} \quad (7.20)$$

Except for the four cases just described,  $\Pr(Y = y|X = x)$  is zero for all other values of  $x$  and  $y$  (regardless of the value of the parameters).

**M-step.** We now need to maximize the expectation of  $\log \Pr(X, Y; \theta')$  (which will be a function in terms of the three parameter variables) under the  $q$ -distribution we computed in the E step. The non-zero terms in the expectation are as follows:

$x$	$y$	$q(X = x, Y = y; \theta^{(i)})$	$\log \Pr(X = x, Y = y; \theta')$
$a$	0	$N_a/N$	$\log(1 - p'_0) + \log(1 - p'_1)$
$b$	1	$N_b/N \cdot \Pr(1 b; \theta^{(i)})$	$\log(1 - p'_0) + \log p'_1$
$b$	2	$N_b/N \cdot \Pr(2 b; \theta^{(i)})$	$\log p'_0 + \log(1 - p'_2)$
$c$	3	$N_c/N$	$\log p'_0 + \log p'_2$

Multiplying across each row and adding from top to bottom yields the expectation we wish to maximize. Each parameter can be optimized independently using differentiation. The resulting optimal values are expressed in terms of the counts in  $\mathbf{x}$  and  $\theta^{(i)}$ :

$$p_0 = \frac{\Pr(2|b; \theta^{(i)}) \cdot N_b + N_c}{N} \quad (7.21)$$

$$p_1 = \frac{\Pr(1|b; \theta^{(i)}) \cdot N_b}{N_a + \Pr(1|b; \theta^{(i)}) \cdot N_b} \quad (7.22)$$

$$p_2 = \frac{N_c}{\Pr(2|b; \theta^{(i)}) \cdot N_b + N_c} \quad (7.23)$$

It is worth noting that the form of these expressions is quite similar to the fully observed maximum likelihood estimate. However, rather than depending on *exact* path counts, the statistics used are the *expected* path counts, given  $\mathbf{x}$  and parameters  $\theta^{(i)}$ .

Typically, the values computed at the end of the M-step would serve as new parameters for another iteration of EM. However, the example we have presented here is quite simple and the model converges to a global optimum after a single iteration. For most models, EM requires several iterations to converge, and it may not find a global optimum. And since EM only finds a locally optimal solution, the final parameter values depend on the values chose for  $\theta^{(0)}$ .

## 7.2 Hidden Markov Models

To give a more substantial and useful example of models whose parameters may be estimated using EM, we turn to hidden Markov models (HMMs). HMMs are models of data that are ordered *sequentially* (temporally, from left to right, etc.), such as words in a sentence, base pairs in a gene, or letters in a word. These simple but powerful models have been used in applications as diverse as speech recognition [78], information extraction [139], gene finding [143], part of speech tagging [44], stock market forecasting [70], text retrieval [108], and word alignment of parallel (translated) texts [150] (more in Section 7.4).

In an HMM, the data being modeled is posited to have been generated from an underlying *Markov process*, which is a stochastic process consisting of a finite set of states where the probability of entering a state at time  $t + 1$  depends only on the state of the process at time  $t$  [130]. Alternatively, one can view a Markov process as a probabilistic variant of a finite state machine, where transitions are taken probabilistically. As another point of comparison, the PageRank algorithm considered in the previous chapter (Section 5.3) can be understood as a Markov process: the probability of following any link on a particular page is independent of the path taken to reach that page. The states of this Markov process are, however, not directly observable (i.e., *hidden*). Instead, at each time step, an observable token (e.g., a word, base pair, or letter) is emitted according to a probability distribution conditioned on the identity of the state that the underlying process is in.

A hidden Markov model  $\mathcal{M}$  is defined as a tuple  $\langle \mathcal{S}, \mathcal{O}, \theta \rangle$ .  $\mathcal{S}$  is a finite set of states, which generate symbols from a finite observation vocabulary  $\mathcal{O}$ . Following convention, we assume that variables  $q$ ,  $r$ , and  $s$  refer to states in  $\mathcal{S}$ , and  $o$  refers to symbols in the observation vocabulary  $\mathcal{O}$ . This model is parameterized by the tuple  $\theta = \langle A, B, \pi \rangle$  consisting of an  $|\mathcal{S}| \times |\mathcal{S}|$  matrix  $A$  of *transition probabilities*, where  $A_q(r)$  gives the probability of transitioning from state  $q$  to state  $r$ ; an  $|\mathcal{S}| \times |\mathcal{O}|$  matrix  $B$  of *emission probabilities*, where  $B_q(o)$  gives the probability that symbol  $o$  will be emitted from state  $q$ ; and an  $|\mathcal{S}|$ -dimensional vector  $\pi$ , where  $\pi_q$  is the probability that the process starts in state  $q$ .<sup>7</sup> These matrices may be dense, but for many applications sparse parameterizations are useful. We further stipulate that  $A_q(r) \geq 0$ ,  $B_q(o) \geq 0$ , and  $\pi_q \geq 0$  for all  $q$ ,  $r$ , and  $o$ , as well as that:

$$\sum_{r \in \mathcal{S}} A_q(r) = 1 \quad \forall q \quad \sum_{o \in \mathcal{O}} B_q(o) = 1 \quad \forall q \quad \sum_{q \in \mathcal{S}} \pi_q = 1 \quad (7.24)$$

A sequence of observations of length  $\tau$  is generated as follows:

Step 0, let  $t = 1$  and select an initial state  $q$  according to the distribution  $\pi$ .

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<sup>7</sup>This is only one possible definition of an HMM, but it is one that is useful for many text processing problems. In alternative definitions, initial and final states may be handled differently, observations may be emitted during the transition between states, or continuous-valued observations may be emitted (for example, from a Gaussian distribution).

Step 1, an observation symbol from  $\mathcal{O}$  is emitted according to the distribution  $B_q$ .

Step 2, a new  $q$  is drawn according to the distribution  $A_q$ .

Step 3,  $t$  is incremented, and if  $t \leq \tau$ , the process repeats from Step 1.

Since all events generated by this process are conditionally independent, the joint probability of this sequence of observations and the state sequence used to generate it is the product of the individual event probabilities.

Figure 7.3 shows a simple example of a hidden Markov model for part-of-speech tagging, which is the task of assigning to each word in an input sentence its grammatical category (one of the first steps in analyzing textual content). States  $\mathcal{S} = \{\text{DET}, \text{ADJ}, \text{NN}, \text{V}\}$  correspond to the parts of speech (determiner, adjective, noun, and verb), and observations  $\mathcal{O} = \{\text{the}, \text{a}, \text{green}, \dots\}$  are a subset of English words. This example illustrates a key intuition behind many applications of HMMs: states correspond to equivalence classes or clustering of observations, and a single observation type may be associated with several clusters (in this example, the word `watch` can be generated by an NN or V, since `watch` can either be a noun or a verb).

### Three Questions for Hidden Markov Models

There are three fundamental questions associated with hidden Markov models:<sup>8</sup>

1. Given a model  $\mathcal{M} = \langle \mathcal{S}, \mathcal{O}, \theta \rangle$ , and an observation sequence of symbols from  $\mathcal{O}$ ,  $\mathbf{x} = \langle x_1, x_2, \dots, x_\tau \rangle$ , what is the probability that  $\mathcal{M}$  generated the data (summing over all possible state sequences,  $\mathcal{Y}$ )?

$$\Pr(\mathbf{x}) = \sum_{\mathbf{y} \in \mathcal{Y}} \Pr(\mathbf{x}, \mathbf{y}; \theta) \quad (7.25)$$

2. Given a model  $\mathcal{M} = \langle \mathcal{S}, \mathcal{O}, \theta \rangle$  and an observation sequence  $\mathbf{x}$ , what is the most likely sequence of states that generated the data?

$$\mathbf{y}^* = \arg \max_{\mathbf{y} \in \mathcal{Y}} \Pr(\mathbf{x}, \mathbf{y}; \theta) \quad (7.26)$$

3. Given a set of states  $\mathcal{S}$ , an observation vocabulary  $\mathcal{O}$ , and a series of  $\ell$  i.i.d. observation sequences  $\langle \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_\ell \rangle$ , what are the parameters  $\theta = \langle A, B, \pi \rangle$  that maximize the likelihood of the training data?

$$\theta^* = \arg \max_{\theta} \prod_{i=1}^{\ell} \sum_{\mathbf{y} \in \mathcal{Y}} \Pr(\mathbf{x}_i, \mathbf{y}; \theta) \quad (7.27)$$

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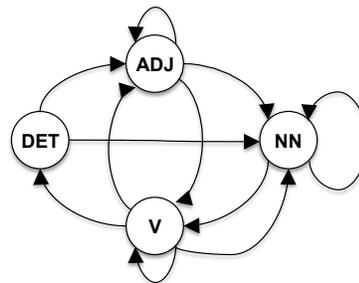
<sup>8</sup>The organization of this section is based in part on ideas from Lawrence Rabiner's HMM tutorial [125].

**Initial Probabilities:**

<b>DET</b>	<b>ADJ</b>	<b>NN</b>	<b>V</b>
0.5	0.1	0.3	0.1

**Transition Probabilities:**

	<b>DET</b>	<b>ADJ</b>	<b>NN</b>	<b>V</b>
<b>DET</b>	0.0	0.0	0.0	0.5
<b>ADJ</b>	0.3	0.2	0.1	0.2
<b>NN</b>	0.7	0.7	0.4	0.2
<b>V</b>	0.0	0.1	0.5	0.1



**Emission Probabilities:**

<b>DET</b>		<b>ADJ</b>		<b>NN</b>		<b>V</b>	
the	0.7	green	0.1	book	0.3	might	0.2
a	0.3	big	0.4	plants	0.2	watch	0.3
		old	0.4	people	0.2	watches	0.2
		might	0.1	person	0.1	loves	0.1
				John	0.1	reads	0.19
				watch	0.1	books	0.01

**Examples:**

John	might	watch			
<b>NN</b>	<b>V</b>	<b>V</b>			
the	old	person	loves	big	books
<b>DET</b>	<b>ADJ</b>	<b>NN</b>	<b>V</b>	<b>ADJ</b>	<b>NN</b>

Figure 7.3: An example HMM that relates part-of-speech tags to vocabulary items in an English-like language. Possible (probability > 0) transitions for the Markov process are shown graphically. In the example outputs, the state sequences corresponding to the emissions are written beneath the emitted symbols.

Using our definition of an HMM, the answers to the first two questions are in principle quite trivial to compute: by iterating over all state sequences  $\mathcal{Y}$ , the probability that each generated  $\mathbf{x}$  can be computed by looking up and multiplying the relevant probabilities in  $A$ ,  $B$ , and  $\pi$ , and then summing the result or taking the maximum. And, as we hinted at in the previous section, the third question can be answered using EM. Unfortunately, even with all the distributed computing power MapReduce makes available, we will quickly run into trouble if we try to use this naïve strategy since there are  $|\mathcal{S}|^\tau$  distinct state sequences of length  $\tau$ , making exhaustive enumeration computationally intractable. Fortunately, because the underlying model behaves exactly the same whenever it is in some state, regardless of how it got to that state, we can use dynamic programming algorithms to answer all of the above questions without summing over exponentially many sequences.

## The Forward Algorithm

Given some observation sequence, for example  $\mathbf{x} = \langle \text{John, might, watch} \rangle$ , Question 1 asks what is the probability that this sequence was generated by an HMM  $\mathcal{M} = \langle \mathcal{S}, \mathcal{O}, \theta \rangle$ . For the purposes of illustration, we assume that  $\mathcal{M}$  is defined as shown in Figure 7.3.

There are two ways to compute the probability of  $\mathbf{x}$  having been generated by  $\mathcal{M}$ . The first is to compute the sum over the joint probability of  $\mathbf{x}$  and every possible labeling  $\mathbf{y}' \in \{ \langle \text{DET, DET, DET} \rangle, \langle \text{DET, DET, NN} \rangle, \langle \text{DET, DET, V} \rangle, \dots \}$ . As indicated above this is not feasible for most sequences, since the set of possible labels is exponential in the length of  $\mathbf{x}$ . The second, fortunately, is much more efficient.

We can make use of what is known as the *forward algorithm* to compute the desired probability in polynomial time. We assume a model  $\mathcal{M} = \langle \mathcal{S}, \mathcal{O}, \theta \rangle$  as defined above. This algorithm works by recursively computing the answer to a related question: what is the probability that the process is in state  $q$  at time  $t$  and has generated  $\langle x_1, x_2, \dots, x_t \rangle$ ? Call this probability  $\alpha_t(q)$ . Thus,  $\alpha_t(q)$  is a two dimensional matrix (of size  $|\mathbf{x}| \times |\mathcal{S}|$ ), called a *trellis*. It is easy to see that the values of  $\alpha_1(q)$  can be computed as the product of two independent probabilities: the probability of starting in state  $q$  and the probability of state  $q$  generating  $x_1$ :

$$\alpha_1(q) = \pi_q \cdot B_q(x_1) \tag{7.28}$$

From this, it's not hard to see that the values of  $\alpha_2(r)$  for every  $r$  can be computed in terms of the  $|\mathcal{S}|$  values in  $\alpha_1(\cdot)$  and the observation  $x_2$ :

$$\alpha_2(r) = B_r(x_2) \cdot \sum_{q \in \mathcal{S}} \alpha_1(q) \cdot A_q(r) \tag{7.29}$$

This works because there are  $|\mathcal{S}|$  different ways to get to state  $r$  at time  $t = 2$ : starting from state  $1, 2, \dots, |\mathcal{S}|$  and transitioning to state  $r$ . Furthermore, because the behavior of a Markov process is determined only by the state it is

in at some time (not by how it got to that state),  $\alpha_t(r)$  can always be computed in terms of the  $|\mathcal{S}|$  values in  $\alpha_{t-1}(\cdot)$  and the observation  $x_t$ :

$$\alpha_t(r) = B_r(x_t) \cdot \sum_{q \in \mathcal{S}} \alpha_{t-1}(q) \cdot A_q(r) \quad (7.30)$$

We have now shown how to compute the probability of being in any state  $q$  at any time  $t$ , having generated  $\langle x_1, x_2, \dots, x_t \rangle$ , with the forward algorithm. The probability of the full sequence is the probability of being in time  $|\mathbf{x}|$  and in *any* state, so the answer to Question 1 can be computed simply by summing over  $\alpha$  values at time  $|\mathbf{x}|$  for all states:

$$\Pr(\mathbf{x}; \theta) = \sum_{q \in \mathcal{S}} \alpha_{|\mathbf{x}|}(q) \quad (7.31)$$

In summary, there are two ways of computing the probability that a sequence of observations  $\mathbf{x}$  was generated by  $\mathcal{M}$ : exhaustive enumeration with summing and the forward algorithm. Figure 7.4 illustrates the two possibilities. The upper panel shows the naïve exhaustive approach, enumerating all  $4^3$  possible labels  $\mathbf{y}'$  of  $\mathbf{x}$  and computing their joint probability  $\Pr(\mathbf{x}, \mathbf{y}')$ . Summing over all  $\mathbf{y}'$ , the marginal probability of  $\mathbf{x}$  is found to be 0.00018. The lower panel shows the forward trellis, consisting of  $4 \times 3$  cells. Summing over the final column also yields 0.00018, the same result.

## The Viterbi Algorithm

Given an observation sequence  $\mathbf{x}$ , the second question we might want to ask of  $\mathcal{M}$  is: what is the most likely sequence of states that generated the observations? As with the previous question, the naïve approach to solving this problem is to enumerate all possible labels and find the one with the highest joint probability. Continuing with the example observation sequence  $\mathbf{x} = \langle \text{John, might, watch} \rangle$ , examining the chart of probabilities in the upper panel of Figure 7.4 shows that  $\mathbf{y}^* = \langle \text{NN, V, V} \rangle$  is the most likely sequence of states under our example HMM.

However, a more efficient answer to Question 2 can be computed using the same intuition in the forward algorithm: determine the best state sequence for a short sequence and extend this to easily compute the best sequence for longer ones. This is known as the *Viterbi algorithm*. We define  $\gamma_t(q)$ , the Viterbi probability, to be the most probable sequence of states ending in state  $q$  at time  $t$  and generating observations  $\langle x_1, x_2, \dots, x_t \rangle$ . Since we wish to be able to reconstruct the sequence of states, we define  $bp_t(q)$ , the “backpointer”, to be the state used in this sequence at time  $t - 1$ . The base case for the recursion is as follows (the state index of  $-1$  is used as a placeholder since there is no previous best state at time  $t = 1$ ):

$$\gamma_1(q) = \pi_q \cdot B_q(x_1) \quad (7.32)$$

$$bp_1(q) = -1 \quad (7.33)$$

John	might	watch	$\Pr(x, y)$	John	might	watch	$\Pr(x, y)$	John	might	watch	$\Pr(x, y)$	John	might	watch	$\Pr(x, y)$
DET	DET	DET	0.0	ADJ	DET	DET	0.0	NN	DET	DET	0.0	V	DET	DET	0.0
DET	DET	ADJ	0.0	ADJ	DET	ADJ	0.0	NN	DET	ADJ	0.0	V	DET	ADJ	0.0
DET	DET	NN	0.0	ADJ	DET	NN	0.0	NN	DET	NN	0.0	V	DET	NN	0.0
DET	DET	V	0.0	ADJ	DET	V	0.0	NN	DET	V	0.0	V	DET	V	0.0
DET	ADJ	DET	0.0	ADJ	ADJ	DET	0.0	NN	ADJ	DET	0.0	V	ADJ	DET	0.0
DET	ADJ	ADJ	0.0	ADJ	ADJ	ADJ	0.0	NN	ADJ	ADJ	0.0	V	ADJ	ADJ	0.0
DET	ADJ	NN	0.0	ADJ	ADJ	NN	0.0	NN	ADJ	NN	0.000021	V	ADJ	NN	0.0
DET	ADJ	V	0.0	ADJ	ADJ	V	0.0	NN	ADJ	V	0.000009	V	ADJ	V	0.0
DET	NN	DET	0.0	ADJ	NN	DET	0.0	NN	NN	DET	0.0	V	NN	DET	0.0
DET	NN	ADJ	0.0	ADJ	NN	ADJ	0.0	NN	NN	ADJ	0.0	V	NN	ADJ	0.0
DET	NN	NN	0.0	ADJ	NN	NN	0.0	NN	NN	NN	0.0	V	NN	NN	0.0
DET	NN	V	0.0	ADJ	NN	V	0.0	NN	NN	V	0.0	V	NN	V	0.0
DET	V	DET	0.0	ADJ	V	DET	0.0	NN	V	DET	0.0	V	V	DET	0.0
DET	V	ADJ	0.0	ADJ	V	ADJ	0.0	NN	V	ADJ	0.0	V	V	ADJ	0.0
DET	V	NN	0.0	ADJ	V	NN	0.0	NN	V	NN	0.00006	V	V	NN	0.0
DET	V	V	0.0	ADJ	V	V	0.0	NN	V	V	0.00009	V	V	V	0.0

$$\Pr(\mathbf{x}) = \sum_{\mathbf{y} \in \mathcal{Y}} \Pr(\mathbf{x}, \mathbf{y}; \theta) = 0.00018$$

	John	might	watch
DET	0.0	0.0	0.0
ADJ	0.0	0.0003	0.0
NN	0.03	0.0	0.000081
V	0.0	0.003	0.000099
	$\alpha_1$	$\alpha_2$	$\alpha_3$

$$\Pr(\mathbf{x}) = \sum_{q \in \mathcal{S}} \alpha_3(q) = 0.00018$$

Figure 7.4: Computing the probability of the sequence  $\langle \text{John}, \text{might}, \text{watch} \rangle$  under the HMM given in Figure 7.3 by explicitly summing over all possible sequence labels (top) and using the forward algorithm (bottom).

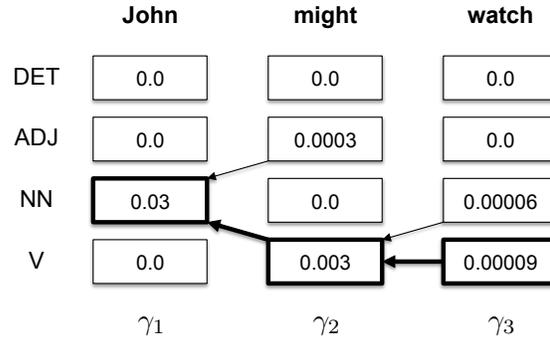


Figure 7.5: Computing the most likely state sequence that generated  $\langle \text{John, might, watch} \rangle$  under the HMM given in Figure 7.3 using the Viterbi algorithm. The most likely state sequence is highlighted in bold and could be recovered programmatically by following backpointers from the maximal probability cell in the last column to the first column (thicker arrows).

The recursion is similar to that of the forward algorithm, except rather than summing over previous states, the *maximum* value of all possible trajectories into state  $r$  at time  $t$  is computed. Note that the backpointer simply records the index of the originating state—a separate computation is not necessary.

$$\gamma_t(r) = \max_{q \in \mathcal{S}} \gamma_{t-1}(q) \cdot A_q(r) \cdot B_r(x_t) \quad (7.34)$$

$$bp_t(r) = \arg \max_{q \in \mathcal{S}} \gamma_{t-1}(q) \cdot A_q(r) \cdot B_r(x_t) \quad (7.35)$$

To compute the best sequence of states,  $\mathbf{y}^*$ , the state with the highest probability path at time  $|\mathbf{x}|$  is selected, and then the backpointers are followed, recursively, to construct the rest of the sequence:

$$y_{|\mathbf{x}|}^* = \arg \max_{q \in \mathcal{S}} \gamma_{|\mathbf{x}|}(q) \quad (7.36)$$

$$y_{t-1}^* = bp_t(y_t) \quad (7.37)$$

Figure 7.5 illustrates a Viterbi trellis, including backpointers that have been used to compute the most likely state sequence.

## Parameter Estimation for HMMs

We now turn to Question 3: given a set of states  $\mathcal{S}$  and observation vocabulary  $\mathcal{O}$ , what are the parameters  $\theta^* = \langle A, B, \pi \rangle$  that maximize the likelihood of a set of training examples,  $\langle \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_\ell \rangle$ ?<sup>9</sup> Since our model is constructed in

<sup>9</sup>Since an HMM models sequences, its training data consists of a collection of example sequences.

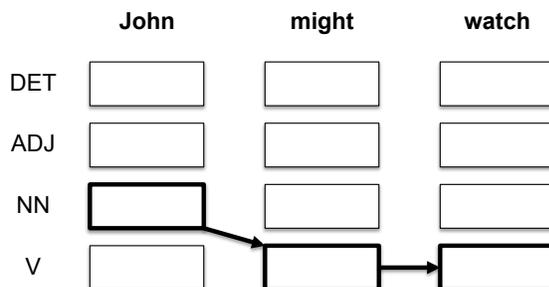


Figure 7.6: A “fully observable” HMM training instance. The output sequence is at the top of the figure, and the corresponding states and transitions are shown in the trellis below.

terms of variables whose values we cannot observe (the state sequence) in the training data, we may train it to optimize the marginal likelihood (summing over *all* state sequences) of  $\mathbf{x}$  using EM. Deriving the EM update equations requires only the application of the techniques presented earlier in this chapter and some differential calculus. However, since the formalism is cumbersome, we will skip a detailed derivation, but readers interested in more information can find it in the relevant citations [78, 125].

In order to make the update equations as intuitive as possible, consider a fully observable HMM, that is, one where both the emissions and the state sequence are observable in all  $\ell$  training instances. In this case, a training instance can be depicted as shown in Figure 7.6. When this is the case, such as when we have a corpus of sentences in which all words have already been tagged with their parts of speech, the maximum likelihood estimate for the parameters can be computed in terms of the counts of the number of times the process transitions from state  $q$  to state  $r$  in all training instances,  $T(q \rightarrow r)$ ; the number of times that state  $q$  emits symbol  $o$ ,  $O(q \uparrow o)$ ; and the number of times the process starts in state  $q$ ,  $I(q)$ . In this example, the process starts in state NN; there is one  $\text{NN} \rightarrow \text{V}$  transition and one  $\text{V} \rightarrow \text{V}$  transition. The NN state emits **John** in the first time step, and V state emits **might** and **watch** in the second and third time steps, respectively. We also define  $N(q)$  to be the number of times the process enters state  $q$ . The maximum likelihood estimates of the parameters in the fully observable case are:

$$\pi_q = \frac{I(q)}{\ell = \sum_r I(r)} \quad (7.38)$$

$$A_q(r) = \frac{T(q \rightarrow r)}{N(q) = \sum_{r'} T(q \rightarrow r')} \quad (7.39)$$

$$B_q(o) = \frac{O(q \uparrow o)}{N(q) = \sum_{o'} O(q \uparrow o')} \quad (7.40)$$

For example, to compute the emission parameters from state NN, we simply need to keep track of the number of times the process is in state NN and what symbol it generates at each of these times. Transition probabilities are computed similarly: to compute, for example, the distribution  $A_{\text{DET}}(\cdot)$ , that is, the probabilities of transitioning away from state DET, we count the number of times the process is in state DET, and keep track of what state the process transitioned into at the next time step. This counting and normalizing be accomplished using the exact same counting and relative frequency algorithms that we described in Section 3.3. Thus, in the fully observable case, parameter estimation is not a new algorithm at all, but one we have seen before.

How should the model parameters be estimated when the state sequence is not provided? It turns out that the update equations have the satisfying form where the optimal parameter values for iteration  $i + 1$  are expressed in terms of the *expectations* of the counts referenced in the fully observed case, according to the posterior distribution over the latent variables given the observations  $\mathbf{x}$  and the parameters  $\theta^{(i)}$ :

$$\pi_q = \frac{\mathbb{E}[I(q)]}{\ell} \quad A_q(r) = \frac{\mathbb{E}[T(q \rightarrow r)]}{\mathbb{E}[N(q)]} \quad B_q(o) = \frac{\mathbb{E}[O(q \uparrow o)]}{\mathbb{E}[N(q)]} \quad (7.41)$$

Because of the independence assumptions made in the HMM, the update equations consist of  $2 \cdot |\mathcal{S}| + 1$  independent optimization problems, just as was the case with the ‘observable’ HMM. Solving for the initial state distribution,  $\pi$ , is one problem; there are  $|\mathcal{S}|$  solving for the transition distributions  $A_q(\cdot)$  from each state  $q$ ; and  $|\mathcal{S}|$  solving for the emissions distributions  $B_q(\cdot)$  from each state  $q$ . Furthermore, we note that the following must hold:

$$\mathbb{E}[N(q)] = \sum_{r \in \mathcal{S}} \mathbb{E}[T(q \rightarrow r)] = \sum_{o \in \mathcal{O}} \mathbb{E}[O(q \uparrow o)] \quad (7.42)$$

As a result, the optimization problems (i.e., Equations 7.38–7.40) require completely independent sets of statistics, which we will utilize later to facilitate efficient parallelization in MapReduce.

How can the expectations in Equation 7.41 be understood? In the fully observed training case, between every time step, there is exactly one transition taken and the source and destination states are observable. By progressing through the Markov chain, we can let each transition count as ‘1’, and we can accumulate the total number of times each kind of transition was taken (by each kind, we simply mean the number of times that one state follows another, for example, the number of times NN follows DET). These statistics can then in turn be used to compute the MLE for an ‘observable’ HMM, as described above. However, when the transition sequence is not observable (as is most often the case), we can instead imagine that at each time step, *every* possible transition (there are  $|\mathcal{S}|^2$  of them, and typically  $|\mathcal{S}|$  is quite small) is taken, with a particular probability. The probability used is the *posterior probability* of the transition, given the model and an observation sequence (we describe how to compute this value below). By summing over all the time

steps in the training data, and using this probability as the ‘count’ (rather than ‘1’ as in the observable case), we compute the *expected count* of the number of times a particular transition was taken, given the training sequence. Furthermore, since the training instances are statistically independent, the value of the expectations can be computed by processing each training instance independently and summing the results.

Similarly for the necessary emission counts (the number of times each symbol in  $\mathcal{O}$  was generated by each state in  $\mathcal{S}$ ), we assume that any state could have generated the observation. We must therefore compute the probability of being in every state at each time point, which is then the size of the emission ‘count’. By summing over all time steps we compute the expected count of the number of times that a particular state generated a particular symbol. These two sets of expectations, which are written formally here, are sufficient to execute the M-step.

$$\mathbb{E}[O(q \uparrow o)] = \sum_{i=1}^{|\mathbf{x}|} \Pr(y_i = q | \mathbf{x}; \theta) \cdot \delta(x_i, o) \quad (7.43)$$

$$\mathbb{E}[T(q \rightarrow r)] = \sum_{i=1}^{|\mathbf{x}|-1} \Pr(y_i = q, y_{i+1} = r | \mathbf{x}; \theta) \quad (7.44)$$

**Posterior probabilities.** The expectations necessary for computing the M-step in HMM training are sums of probabilities that a particular transition is taken, given an observation sequence, and that some state emits some observation symbol, given an observation sequence. These are referred to as posterior probabilities, indicating that they are the probability of some event whose distribution we have a prior belief about, after additional evidence has been taken into consideration (here, the model parameters characterize our prior beliefs, and the observation sequence is the evidence). Both posterior probabilities can be computed by combining the forward probabilities,  $\alpha_t(\cdot)$ , which give the probability of reaching some state at time  $t$ , by any path, and generating the observations  $\langle x_1, x_2, \dots, x_t \rangle$ , with *backward probabilities*,  $\beta_t(\cdot)$ , which give the probability of starting in some state at time  $t$  and generating the rest of the sequence  $\langle x_{t+1}, x_{t+2}, \dots, x_{|\mathbf{x}|} \rangle$ , using any sequence of states to do so. The algorithm for computing the backward probabilities is given a bit later. Once the forward and backward probabilities have been computed, the state transition posterior probabilities and the emission posterior probabilities can be written as follows:

$$\Pr(y_i = q | \mathbf{x}; \theta) = \alpha_i(q) \cdot \beta_i(q) \quad (7.45)$$

$$\Pr(y_i = q, y_{i+1} = r | \mathbf{x}; \theta) = \alpha_i(q) \cdot A_q(r) \cdot B_r(x_{i+1}) \cdot \beta_{i+1}(r) \quad (7.46)$$

Equation 7.45 is the probability of being in state  $q$  at time  $i$ , given  $\mathbf{x}$ , and the correctness of the expression should be clear from the definitions of forward and backward probabilities. The intuition for Equation 7.46, the probability of

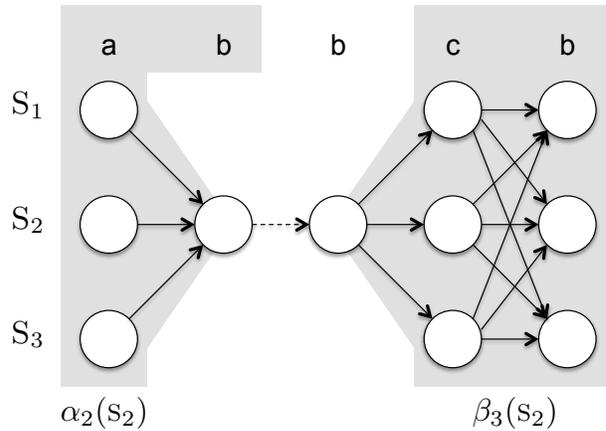


Figure 7.7: Using forward and backward probabilities to compute the posterior probability of the dashed transition, given the observation sequence  $\mathbf{a} \mathbf{b} \mathbf{b} \mathbf{c} \mathbf{b}$ . The shaded area on the left corresponds to the forward probability  $\alpha_2(S_2)$ , and the shaded area on the right corresponds to the backward probability  $\beta_3(S_2)$ .

taking a particular transition at a particular time, is also not complicated: it is the product of four conditionally independent probabilities: the probability of getting to state  $q$  at time  $i$  (having generated the first part of the sequence), the probability of taking transition  $q \rightarrow r$  (which is specified in the parameters,  $\theta$ ), the probability of generating observation  $x_{i+1}$  from state  $r$  (also specified in  $\theta$ ), and the probability of generating the rest of the sequence, along any path. A visualization of the quantities used in computing this probability is shown in Figure 7.7. In this illustration, we assume an HMM with  $\mathcal{S} = \{S_1, S_2, S_3\}$  and  $\mathcal{O} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ .

**The backward algorithm.** Like the forward and Viterbi algorithms introduced above to answer Questions 1 and 2, the backward algorithm uses dynamic programming to incrementally compute  $\beta_t(\cdot)$ . Its base case starts at time  $|\mathbf{x}|$ , and is defined as follows:

$$\beta_{|\mathbf{x}|}(q) = 1 \tag{7.47}$$

To understand the intuition for this base case, keep in mind that since the backward probabilities  $\beta_t(\cdot)$  are the probability of generating the remainder of the sequence *after* time  $t$  (as well as being in some state), and since there is nothing left to generate after time  $|\mathbf{x}|$ , the probability must be 1. The recursion is defined as follows:

$$\beta_t(q) = \sum_{r \in \mathcal{S}} \beta_{t+1}(r) \cdot A_q(r) \cdot B_r(x_{t+1}) \tag{7.48}$$

Unlike the forward and Viterbi algorithms, the backward algorithm is computed from right to left and makes no reference to the start probabilities,  $\pi$ .

### Forward-Backward Training: Summary

In the preceding section, we have shown how to compute all quantities needed to find the parameter settings  $\theta^{(i+1)}$  using EM training with a hidden Markov model  $\mathcal{M} = \langle \mathcal{S}, \mathcal{O}, \theta^{(i)} \rangle$ . To recap: each training instance  $\mathbf{x}$  is processed independently, using the parameter settings of the current iteration,  $\theta^{(i)}$ . For each  $\mathbf{x}$  in the training data, the forward and backward probabilities are computed using the algorithms given above (for this reason, this training algorithm is often referred to as the forward-backward algorithm). The forward and backward probabilities are in turn used to compute the expected number of times the underlying Markov process enters into each state, the number of times each state generates each output symbol type, and the number of times each state transitions into each other state. These expectations are summed over all training instances, completing the E-step. The M-step involves normalizing the expected counts computed in the E-step using the calculations in Equation 7.41, which yields  $\theta^{(i+1)}$ . The process then repeats from the E-step using the new parameters. The number of iterations required for convergence depends on the quality of the initial parameters, and the complexity of the model. For some applications, only a handful of iterations are necessary, whereas for others, hundreds may be required.

Finally, a few practical considerations: HMMs have a non-convex likelihood surface (meaning that it has the equivalent of many hills and valleys in the number of dimensions corresponding to the number of parameters in the model). As a result, EM training is only guaranteed to find a local maximum, and the quality of the learned model may vary considerably, depending on the initial parameters that are used. Strategies for optimal selection of initial parameters depend on the phenomena being modeled. Additionally, if some parameter is assigned a probability of 0 (either as an initial value or during one of the M-step parameter updates), EM will never change this in future iterations. This can be useful, since it provides a way of constraining the structures of the Markov model; however, one must be aware of this behavior.

Another pitfall to avoid when implementing HMMs is arithmetic underflow. HMMs typically define a massive number of sequences, and so the probability of any one of them is often vanishingly small—so small that they often underflow standard floating point representations. A very common solution to this problem is to represent probabilities using their logarithms. Note that expected counts do not typically have this problem and can be represented using normal floating point numbers. See Section 5.4 for additional discussion on working with log probabilities.

### 7.3 EM in MapReduce

Expectation maximization algorithms fit quite naturally into the MapReduce programming model. Although the model being optimized determines the details of the required computations, MapReduce implementations of EM algorithms share a number of characteristics:

- Each iteration of EM is one MapReduce job.
- A controlling process (i.e., driver program) spawns the MapReduce jobs, keeps track of the number of iterations and convergence criteria.
- Model parameters  $\theta^{(i)}$ , which are static for the duration of the MapReduce job, are loaded by each mapper from HDFS or other data provider (e.g., a distributed key-value store).
- Mappers map over independent training instances, computing partial latent variable posteriors (or summary statistics, such as expected counts).
- Reducers sum together the required training statistics *and* solve one or more of the M-step optimization problems.
- Combiners, which sum together the training statistics, are often quite effective at reducing the amount of data that must be written to disk.

The degree of parallelization that can be attained depends on the statistical independence assumed in the model and in the derived quantities required to solve the optimization problems in the M-step. Since parameters are estimated from a collection of samples that are assumed to be i.i.d., the E-step can generally be parallelized effectively since every training instance can be processed independently of the others. In the limit, in fact, each independent training instance could be processed by a separate mapper!<sup>10</sup>

Reducers, however, must aggregate the statistics necessary to solve the optimization problems as required by the model. The degree to which these may be solved independently depends on the structure of the model, and this constrains the number of reducers that may be used. Fortunately, many common models (such as HMMs) require solving several independent optimization problems in the M-step. In this situation, a number of reducers may be run in parallel. Still, it is possible that in the worst case, the M-step optimization problem will not decompose into independent subproblems, making it necessary to use a single reducer.

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<sup>10</sup>Although the wisdom of doing this is questionable, given that the startup costs associated with individual map tasks in Hadoop may be considerable.

## HMM Training in MapReduce

As we would expect, the training of hidden Markov models parallelizes well in MapReduce. The process can be summarized as follows: in each iteration, mappers process training instances, emitting expected event counts computed using the forward-backward algorithm introduced in Section 7.2. Reducers aggregate the expected counts, completing the E-step, and then generate parameter estimates for the next iteration using the updates given in Equation 7.41.

This parallelization strategy is effective for several reasons. First, the majority of the computational effort in HMM training is the running of the forward and backward algorithms. Since there is no limit on the number of mappers that may be run, the full computational resources of a cluster may be brought to bear to solve this problem. Second, since the M-step of an HMM training iteration with  $|\mathcal{S}|$  states in the model consists of  $2 \cdot |\mathcal{S}| + 1$  independent optimization problems that require non-overlapping sets of statistics, this may be exploited with as many as  $2 \cdot |\mathcal{S}| + 1$  reducers running in parallel. While the optimization problem is computationally trivial, being able to reduce in parallel helps avoid the data bottleneck that would limit performance if only a single reducer is used.

The quantities that are required to solve the M-step optimization problem are quite similar to the relative frequency estimation example discussed in Section 3.3; however, rather than counts of observed events, we aggregate *expected* counts of events. As a result of the similarity, we can employ the *stripes* representation for aggregating sets of related values, as described in Section 3.2. A *pairs* approach that requires less memory at the cost of slower performance is also feasible.

**HMM training mapper.** The pseudo-code for the HMM training mapper is given in Figure 7.8. The input consists of key-value pairs with a unique id as the key and a training instance (e.g., a sentence) as the value. For each training instance,  $2n+1$  stripes are emitted with unique keys, and every training instance emits the same set of keys. Each unique key corresponds to one of the independent optimization problems that will be solved in the M-step. The outputs are:

1. the probabilities that the unobserved Markov process begins in each state  $q$ , with a unique key designating that the values are initial state counts;
2. the expected number of times that state  $q$  generated each emission symbol  $o$  (the set of emission symbols included will be just those found in each training instance  $\mathbf{x}$ ), with a key indicating that the associated value is a set of *emission* counts from state  $q$ ; and
3. the expected number of times state  $q$  transitions to each state  $r$ , with a key indicating that the associated value is a set of *transition* counts from state  $q$ .

```

1: class MAPPER
2:   method INITIALIZE(integer iteration)
3:      $\langle \mathcal{S}, \mathcal{O} \rangle \leftarrow \text{READMODEL}$ 
4:      $\theta \leftarrow \langle A, B, \pi \rangle \leftarrow \text{READMODELPARAMS}(iteration)$ 
5:   method MAP(sample id, sequence  $\mathbf{x}$ )
6:      $\alpha \leftarrow \text{FORWARD}(\mathbf{x}, \theta)$  ▷ cf. Section 7.2
7:      $\beta \leftarrow \text{BACKWARD}(\mathbf{x}, \theta)$  ▷ cf. Section 7.2
8:      $I \leftarrow \text{new ASSOCIATIVEARRAY}$  ▷ Initial state expectations
9:     for all  $q \in \mathcal{S}$  do ▷ Loop over states
10:        $I\{q\} \leftarrow \alpha_1(q) \cdot \beta_1(q)$ 
11:      $O \leftarrow \text{new ASSOCIATIVEARRAY of ASSOCIATIVEARRAY}$  ▷ Emissions
12:     for  $t = 1$  to  $|\mathbf{x}|$  do ▷ Loop over observations
13:       for all  $q \in \mathcal{S}$  do ▷ Loop over states
14:          $O\{q\}\{x_t\} \leftarrow O\{q\}\{x_t\} + \alpha_t(q) \cdot \beta_t(q)$ 
15:        $t \leftarrow t + 1$ 
16:      $T \leftarrow \text{new ASSOCIATIVEARRAY of ASSOCIATIVEARRAY}$ 
17: ▷ Transitions
18:     for  $t = 1$  to  $|\mathbf{x}| - 1$  do ▷ Loop over observations
19:       for all  $q \in \mathcal{S}$  do ▷ Loop over states
20:         for all  $r \in \mathcal{S}$  do ▷ Loop over states
21:            $T\{q\}\{r\} \leftarrow T\{q\}\{r\} + \alpha_t(q) \cdot A_q(r) \cdot B_r(x_{t+1}) \cdot \beta_{t+1}(r)$ 
22:          $t \leftarrow t + 1$ 
23:     EMIT(string ‘initial’, stripe  $I$ )
24:     for all  $q \in \mathcal{S}$  do ▷ Loop over states
25:       EMIT(string ‘emit from ’ +  $q$ , stripe  $O\{q\}$ )
26:       EMIT(string ‘transit from ’ +  $q$ , stripe  $T\{q\}$ )

```

Figure 7.8: Mapper pseudo-code for training hidden Markov models using EM. The mappers map over training instances (i.e., sequences of observations  $\mathbf{x}_i$ ) and generate the expected counts of initial states, emissions, and transitions taken to generate the sequence.

**HMM training reducer.** The reducer for one iteration of HMM training, shown together with an optional combiner in Figure 7.9, aggregates the count collections associated with each key by summing them. When the values for each key have been completely aggregated, the associative array contains all of the statistics necessary to compute a subset of the parameters for the next EM iteration. The optimal parameter settings for the following iteration are computed simply by computing the relative frequency of each event with respect to its expected count at the current iteration. The new computed parameters are emitted from the reducer and written to HDFS. Note that they will be spread across  $2 \cdot |\mathcal{S}| + 1$  keys, representing initial state probabilities  $\pi$ , transition probabilities  $A_q$  for each state  $q$ , and emission probabilities  $B_q$  for each state  $q$ .

```

1: class COMBINER
2:   method COMBINE(string  $t$ , stripes [ $C_1, C_2, \dots$ ])
3:      $C_f \leftarrow$  new ASSOCIATIVEARRAY
4:     for all stripe  $C \in$  stripes [ $C_1, C_2, \dots$ ] do
5:       SUM( $C_f, C$ )
6:     EMIT(string  $t$ , stripe  $C_f$ )

1: class REDUCER
2:   method REDUCE(string  $t$ , stripes [ $C_1, C_2, \dots$ ])
3:      $C_f \leftarrow$  new ASSOCIATIVEARRAY
4:     for all stripe  $C \in$  stripes [ $C_1, C_2, \dots$ ] do
5:       SUM( $C_f, C$ )
6:      $z \leftarrow 0$ 
7:     for all  $\langle k, v \rangle \in C_f$  do
8:        $z \leftarrow z + v$ 
9:      $P_f \leftarrow$  new ASSOCIATIVEARRAY ▷ Final parameters vector
10:    for all  $\langle k, v \rangle \in C_f$  do
11:       $P_f\{k\} \leftarrow v/z$ 
12:    EMIT(string  $t$ , stripe  $P_f$ )

```

Figure 7.9: Combiner and reducer pseudo-code for training hidden Markov models using EM. The HMMs considered in this book are fully parameterized by multinomial distributions, so reducers do not require special logic to handle different types of model parameters (since they are all of the same type).

## 7.4 Case Study: Word Alignment for Statistical Machine Translation

To illustrate the real-world benefits of expectation maximization algorithms using MapReduce, we turn to the problem of word alignment, which is an important task in statistical machine translation that is typically solved using models whose parameters are learned with EM.

We begin by giving a brief introduction to statistical machine translation and the phrase-based translation approach; for a more comprehensive introduction, refer to [85, 97]. Fully-automated translation has been studied since the earliest days of electronic computers. After successes with code-breaking during World War II, there was considerable optimism that translation of human languages would be another soluble problem. In the early years, work on translation was dominated by manual attempts to encode linguistic knowledge into computers—another instance of the ‘rule-based’ approach we described in the introduction to this chapter. These early attempts failed to live up to the admittedly optimistic expectations. For a number of years, the idea of fully automated translation was viewed with skepticism. Not only was constructing a translation system labor intensive, but translation pairs had to be developed

independently, meaning that improvements in a Russian-English translation system could not, for the most part, be leveraged to improve a French-English system.

After languishing for a number of years, the field was reinvigorated in the late 1980s when researchers at IBM pioneered the development of *statistical machine translation* (SMT), which took a data-driven approach to solving the problem of machine translation, attempting to improve both the quality of translation while reducing the cost of developing systems [29]. The core idea of SMT is to equip the computer to *learn* how to translate, using example translations which are produced for other purposes, and modeling the process as a statistical process with some parameters  $\theta$  relating strings in a source language (typically denoted as  $\mathbf{f}$ ) to strings in a target language (typically denoted as  $\mathbf{e}$ ):

$$\mathbf{e}^* = \arg \max_{\mathbf{e}} \Pr(\mathbf{e}|\mathbf{f}; \theta) \quad (7.49)$$

With the statistical approach, translation systems can be developed cheaply and quickly for any language pair, as long as there is sufficient training data available. Furthermore, improvements in learning algorithms and statistical modeling can yield benefits in many translation pairs at once, rather than being specific to individual language pairs. Thus, SMT, like many other topics we are considering in this book, is an attempt to leverage the vast quantities of textual data that is available to solve problems that would otherwise require considerable manual effort to encode specialized knowledge. Since the advent of statistical approaches to translation, the field has grown tremendously and numerous statistical models of translation have been developed, with many incorporating quite specialized knowledge about the behavior of natural language as biases in their learning algorithms.

## Statistical Phrase-Based Translation

One approach to statistical translation that is simple yet powerful is called *phrase-based translation* [86]. We provide a rough outline of the process since it is representative of most state-of-the-art statistical translation systems, such as the one used inside Google Translate.<sup>11</sup> Phrase-based translation works by learning how strings of words, called *phrases*, translate between languages.<sup>12</sup> Example phrase pairs for Spanish-English translation might include:

$\langle \textit{los estudiantes}, \textit{the students} \rangle$ ,  $\langle \textit{los estudiantes}, \textit{some students} \rangle$ , and  
 $\langle \textit{soy}, \textit{i am} \rangle$ .

From a few hundred thousand sentences of example translations, many millions of such phrase pairs may be automatically learned.

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<sup>11</sup><http://translate.google.com>

<sup>12</sup>Phrases are simply sequences of words; they are not required to correspond to the definition of a phrase in any linguistic theory.

The starting point is typically a parallel corpus (also called *bitext*), which contains *pairs* of sentences in two languages that are translations of each other. Parallel corpora are frequently generated as the byproduct of an organization's effort to disseminate information in multiple languages, for example, proceedings of the Canadian Parliament in French and English, and text generated by the United Nations in many different languages. The parallel corpus is then annotated with *word alignments*, which indicate which words in one language correspond to words in the other. By using these word alignments as a skeleton, phrases can be extracted from the sentence that is likely to preserve the meaning relationships represented by the word alignment. While an explanation of the process is not necessary here, we mention it as a motivation for learning word alignments, which we show below how to compute with EM. After phrase extraction, each phrase pair is associated with a number of scores which, taken together, are used to compute the phrase translation probability, a conditional probability that reflects how likely the source phrase translates into the target phrase. We briefly note that although EM could be utilized to learn the phrase translation probabilities, this is not typically done in practice since the maximum likelihood solution turns out to be quite bad for this problem. The collection of phrase pairs and their scores are referred to as the *translation model*. In addition to the translation model, phrase-based translation depends on a *language model*, which gives the probability of a string in the target language. The translation model attempts to preserve the meaning of the source language during the translation process, while the language model ensures that the output is fluent and grammatical in the target language. The phrase-based translation process is summarized in Figure 7.10.

A language model gives the probability that a string of words:

$$\mathbf{w} = \langle w_1, w_2, \dots, w_n \rangle,$$

written as  $w_1^n$  for short, is a string in the target language. By the chain rule of probability, we get:

$$\Pr(w_1^n) = \Pr(w_1) \Pr(w_2|w_1) \Pr(w_3|w_1^2) \dots \Pr(w_n|w_1^{n-1}) \quad (7.50)$$

$$= \prod_{k=1}^n \Pr(w_k|w_1^{k-1}) \quad (7.51)$$

Due to the extremely large number of parameters involved in estimating such a model directly, it is customary to make the *Markov assumption*, that the sequence histories only depend on prior local context. That is, an  $n$ -gram language model is equivalent to a  $(n-1)$ th-order Markov model. Thus, we can approximate  $P(w_k|w_1^{k-1})$  as follows:

$$\text{bigrams: } P(w_k|w_1^{k-1}) \approx P(w_k|w_{k-1}) \quad (7.52)$$

$$\text{trigrams: } P(w_k|w_1^{k-1}) \approx P(w_k|w_{k-1}w_{k-2}) \quad (7.53)$$

$$\text{\(n\)-grams: } P(w_k|w_1^{k-1}) \approx P(w_k|w_{k-n+1}^{k-1}) \quad (7.54)$$

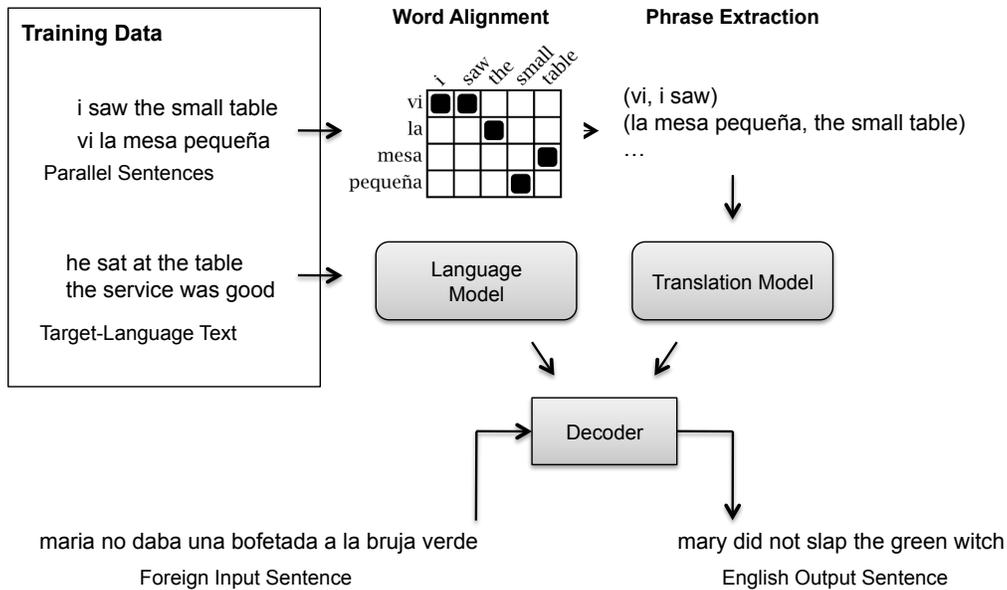


Figure 7.10: The standard phrase-based machine translation architecture. The translation model is constructed with phrases extracted from a word-aligned parallel corpus. The language model is estimated from a monolingual corpus. Both serve as input to the decoder, which performs the actual translation.

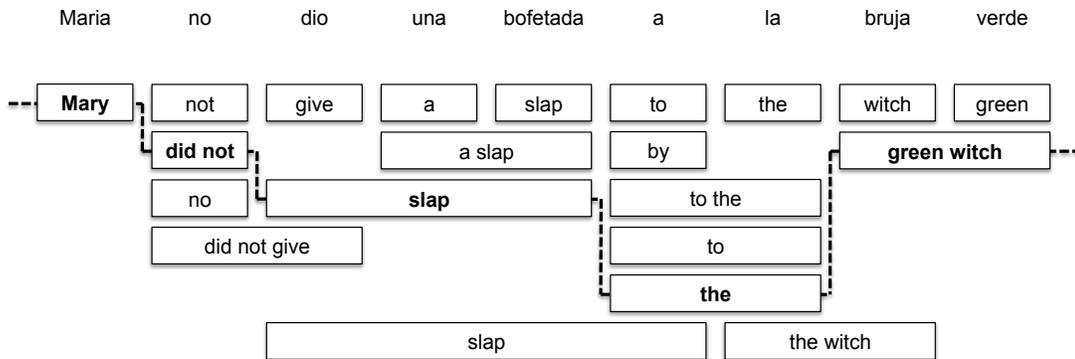


Figure 7.11: Translation coverage of the sentence *Maria no dio una bofetada a la bruja verde* by a phrase-based model. The best possible translation path is indicated with a dashed line.

The probabilities used in computing  $\Pr(w_1^n)$  based on an  $n$ -gram language model are generally estimated from a monolingual corpus of target language text. Since only target language text is necessary (without any additional annotation), language modeling has been well served by large-data approaches that take advantage of the vast quantities of text available on the web.

To translate an input sentence  $\mathbf{f}$ , the phrase-based *decoder* creates a matrix of all translation possibilities of all substrings in the input string, as an example illustrates in Figure 7.11. A sequence of phrase pairs is selected such that each word in  $\mathbf{f}$  is translated exactly once.<sup>13</sup> The decoder seeks to find the translation that maximizes the product of the translation probabilities of the phrases used and the language model probability of the resulting string in the target language. Because the phrase translation probabilities are independent of each other and the Markov assumption made in the language model, this may be done efficiently using dynamic programming. For a detailed introduction to phrase-based decoding, we refer the reader to a recent textbook by Koehn [85].

### Brief Digression: Language Modeling with MapReduce

Statistical machine translation provides the context for a brief digression on distributed parameter estimation for language models using MapReduce, and provides another example illustrating the effectiveness data-driven approaches in general. We briefly touched upon this work in Chapter 1. Even after making the Markov assumption, training  $n$ -gram language models still requires estimating an enormous number of parameters: potentially  $V^n$ , where  $V$  is the number of words in the vocabulary. For higher-order models (e.g., 5-grams) used in real-world applications, the number of parameters can easily exceed the number of words from which to estimate those parameters. In fact, most  $n$ -grams will never be observed in a corpus, no matter how large. To cope with this sparseness, researchers have developed a number of smoothing techniques [102], which all share the basic idea of moving probability mass from observed to unseen events in a principled manner. For many applications, a state-of-the-art approach is known as Kneser-Ney smoothing [35].

In 2007, Brants et al. [25] reported experimental results that answered an interesting question: given the availability of large corpora (i.e., the web), could a simpler smoothing strategy, applied to more text, beat Kneser-Ney in a machine translation task? It should come as no surprise that the answer is *yes*. Brants et al. introduced a technique known as “stupid backoff” that was exceedingly simple and so naïve that the resulting model didn’t even define a valid probability distribution (it assigned arbitrary scores as opposed to probabilities). The simplicity, however, afforded an extremely scalable implementations in MapReduce. With smaller corpora, stupid backoff didn’t work as well as Kneser-Ney in generating accurate and fluent translations. However, as the amount of data increased, the gap between stupid backoff and Kneser-Ney

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<sup>13</sup>The phrases may not necessarily be selected in a strict left-to-right order. Being able to vary the order of the phrases used is necessary since languages may express the same ideas using different word orders.

narrowed, and eventually disappeared with sufficient data. Furthermore, with stupid backoff it was possible to train a language model on more data than was feasible with Kneser-Ney smoothing. Applying this language model to a machine translation task yielded better results than a (smaller) language model trained with Kneser-Ney smoothing.

The role of the language model in statistical machine translation is to select fluent, grammatical translations from a large hypothesis space: the more training data a language model has access to, the better its description of relevant language phenomena and hence its ability to select good translations. Once again, large data triumphs! For more information about estimating language models using MapReduce, we refer the reader to a forthcoming book from Morgan & Claypool [26].

## Word Alignment

Word alignments, which are necessary for building phrase-based translation models (as well as many other more sophisticated translation models), can be learned automatically using EM. In this section, we introduce a popular alignment model based on HMMs.

In the statistical model of word alignment considered here, the observable variables are the words in the source and target sentences (conventionally written using the variables  $\mathbf{f}$  and  $\mathbf{e}$ , respectively), and their alignment is a latent variable. To make this model tractable, we assume that words are translated independently of one another, which means that the model’s parameters include the probability of any word in the source language translating to any word in the target language. While this independence assumption is problematic in many ways, it results in a simple model structure that admits efficient inference yet produces reasonable alignments. Alignment models that make this assumption generate a string  $\mathbf{e}$  in the target language by selecting words in the source language according to a lexical translation distribution. The indices of the words in  $\mathbf{f}$  used to generate each word in  $\mathbf{e}$  are stored in an alignment variable,  $\mathbf{a}$ .<sup>14</sup> This means that the variable  $a_i$  indicates the source word position of the  $i^{\text{th}}$  target word generated, and  $|\mathbf{a}| = |\mathbf{e}|$ . Using these assumptions, the probability of an alignment and translation can be written as follows:

$$\Pr(\mathbf{e}, \mathbf{a} | \mathbf{f}) = \underbrace{\Pr(\mathbf{a} | \mathbf{f}, \mathbf{e})}_{\text{Alignment probability}} \times \underbrace{\prod_{i=1}^{|\mathbf{e}|} \Pr(e_i | f_{a_i})}_{\text{Lexical probability}} \quad (7.55)$$

Since we have parallel corpora consisting of only  $\langle \mathbf{f}, \mathbf{e} \rangle$  pairs, we can learn the parameters for this model using EM and treating  $\mathbf{a}$  as a latent variable. However, to combat data sparsity in the alignment probability, we must make some

<sup>14</sup>In the original presentation of statistical lexical translation models, a special null word is added to the source sentences, which permits words to be inserted ‘out of nowhere’. Since this does not change any of the important details of training, we omit it from our presentation for simplicity.

further simplifying assumptions. By letting the probability of an alignment depend only on the position of the previous aligned word we capture a valuable insight (namely, words that are nearby in the source language will tend to be nearby in the target language), and our model acquires the structure of an HMM [150]:

$$\Pr(\mathbf{e}, \mathbf{a}|\mathbf{f}) = \underbrace{\prod_{i=1}^{|\mathbf{e}|} \Pr(a_i|a_{i-1})}_{\text{Transition probability}} \times \underbrace{\prod_{i=1}^{|\mathbf{e}|} \Pr(e_i|f_{a_i})}_{\text{Emission probability}} \quad (7.56)$$

This model can be trained using the forward-backward algorithm described in the previous section, summing over all settings of  $\mathbf{a}$ , and the best alignment for a sentence pair can be found using the Viterbi algorithm.

To properly initialize this HMM, it is conventional to further simplify the alignment probability model, and use this simpler model to learn initial lexical translation (emission) parameters for the HMM. The favored simplification is to assert that all alignments are uniformly probable:

$$\Pr(\mathbf{e}, \mathbf{a}|\mathbf{f}) = \frac{1}{|\mathbf{f}|^{|\mathbf{e}|}} \times \prod_{i=1}^{|\mathbf{e}|} \Pr(e_i|f_{a_i}) \quad (7.57)$$

This model is known as IBM Model 1. It is attractive for initialization because it is convex everywhere, and therefore EM will learn the same solution regardless of initialization. Finally, while the forward-backward algorithm could be used to compute the expected counts necessary for training this model by setting  $A_q(r)$  to be a constant value for all  $q$  and  $r$ , the uniformity assumption means that the expected emission counts can be estimated in time  $O(|\mathbf{e}| \cdot |\mathbf{f}|)$ , rather than time  $O(|\mathbf{e}| \cdot |\mathbf{f}|^2)$  required by the forward-backward algorithm.

## Experiments

How well does a MapReduce word aligner for statistical machine translation perform? We describe previously-published results [54] that compared a Java-based Hadoop implementation against a highly optimized word aligner called Giza++ [112], which was written in C++ and designed to run efficiently on a single core. We compared the training time of Giza++ and our aligner on a Hadoop cluster with 19 slave nodes, each with two single-core processors and two disks (38 cores total).

Figure 7.12 shows the performance of Giza++ in terms of the running time of a single EM iteration for both Model 1 and the HMM alignment model as a function of the number of training pairs. Both axes in the figure are on a log scale, but the ticks on the  $y$ -axis are aligned with ‘meaningful’ time intervals rather than exact orders of magnitude. There are three things to note. First, the running time scales linearly with the size of the training data. Second, the HMM is a constant factor slower than Model 1. Third, the alignment process

is quite slow as the size of the training data grows—at one million sentences, a single iteration takes over three hours to complete! Five iterations are generally necessary to train the models, which means that full training takes the better part of a day.

In Figure 7.13 we plot the running time of our MapReduce implementation running on the 38-core cluster described above. For reference, we plot points indicating what 1/38 of the running time of the Giza++ iterations would be at each data size, which gives a rough indication of what an ‘ideal’ parallelization could achieve, assuming that there was no overhead associated with distributing computation across these machines. Three things may be observed in the results. First, as the amount of data increases, the relative cost of the overhead associated with distributing data, marshaling and aggregating counts, decreases. At one million sentence pairs of training data, the HMM alignment iterations begin to approach optimal runtime efficiency. Second, Model 1, which we observe is light on computation, does not approach the theoretical performance of an ideal parallelization, and in fact, has almost the same running time as the HMM alignment algorithm. We conclude that the overhead associated with distributing and aggregating data is significant compared to the Model 1 computations, although a comparison with Figure 7.12 indicates that the MapReduce implementation is still substantially faster than the single core implementation, at least once a certain training data size is reached. Finally, we note that, in comparison to the running times of the single-core implementation, at large data sizes, there is a significant advantage to using the distributed implementation, even of Model 1.

Although these results do confound several variables (Java vs. C++ performance, memory usage patterns), it is reasonable to expect that the confounds would tend to make the single-core system’s performance appear relatively *better* than the MapReduce system (which is, of course, the opposite pattern from what we actually observe). Furthermore, these results show that when computation is distributed over a cluster of many machines, even an unsophisticated implementation of the HMM aligner could compete favorably with a highly optimized single-core system whose performance is well-known to many people in the MT research community.

Why are these results important? Perhaps the most significant reason is that the quantity of parallel data that is available to train statistical machine translation models is ever increasing, and as is the case with so many problems we have encountered, more data leads to improvements in translation quality [54]. Recently a corpus of one billion words of French-English data was mined automatically from the web and released publicly [33].<sup>15</sup> Single-core solutions to model construction simply cannot keep pace with the amount of translated data that is constantly being produced. Fortunately, several independent researchers have shown that existing modeling algorithms can be expressed naturally and effectively using MapReduce, which means that we can take advantage of this data. Furthermore, the results presented here show that

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<sup>15</sup> <http://www.statmt.org/wmt10/translation-task.html>

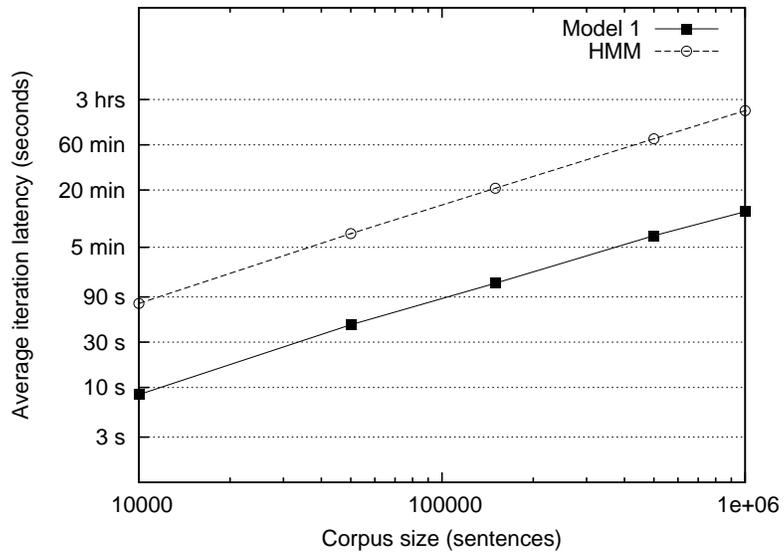


Figure 7.12: Running times of Giza++ (baseline single-core system) for Model 1 and HMM training iterations at various corpus sizes.

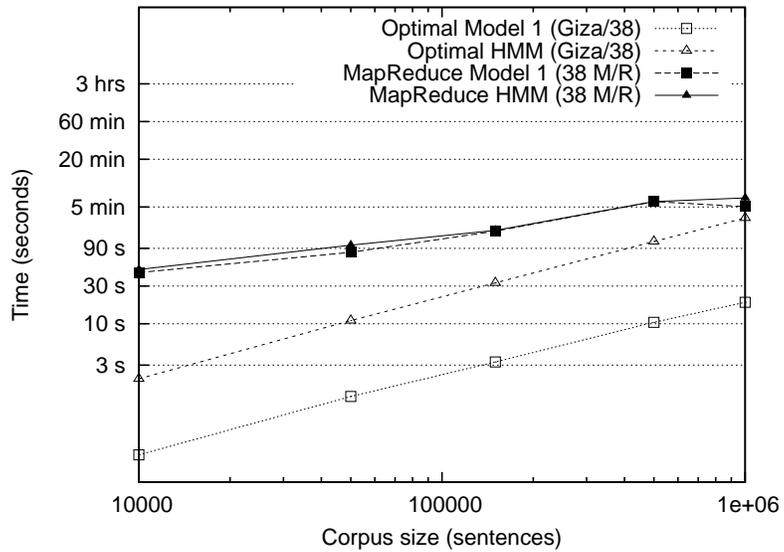


Figure 7.13: Running times of our MapReduce implementation of Model 1 and HMM training iterations at various corpus sizes. For reference, 1/38 running times of the Giza++ models are shown.

even at data sizes that may be tractable on single machines, significant performance improvements are attainable using MapReduce implementations. This improvement reduces experimental turnaround times, which allows researchers to more quickly explore the solution space—which will, we hope, lead to rapid new developments in statistical machine translation.

For the reader interested in statistical machine translation, there is an open source Hadoop-based MapReduce implementation of a training pipeline for phrase-based translation that includes word alignment, phrase extraction, and phrase scoring [56].

## 7.5 EM-Like Algorithms

This chapter has focused on expectation maximization algorithms and their implementation in the MapReduce programming framework. These important algorithms are indispensable for learning models with latent structure from unannotated data, and they can be implemented quite naturally in MapReduce. We now explore some related learning algorithms that are similar to EM but can be used to solve more general problems, and discuss their implementation.

In this section we focus on *gradient-based optimization*, which refers to a class of techniques used to optimize any objective function, provided it is differentiable with respect to the parameters being optimized. Gradient-based optimization is particularly useful in the learning of maximum entropy (maxent) models [110] and conditional random fields (CRF) [87] that have an exponential form and are trained to maximize conditional likelihood. In addition to being widely used supervised classification models in text processing (meaning that during training, both the data and their annotations must be observable), their gradients take the form of expectations. As a result, some of the previously-introduced techniques are also applicable for optimizing these models.

### Gradient-Based Optimization and Log-Linear Models

Gradient-based optimization refers to a class of iterative optimization algorithms that use the derivatives of a function to find the parameters that yield a minimal or maximal value of that function. Obviously, these algorithms are only applicable in cases where a useful objective exists, is differentiable, and its derivatives can be efficiently evaluated. Fortunately, this is the case for many important problems of interest in text processing. For the purposes of this discussion, we will give examples in terms of minimizing functions.

Assume that we have some real-valued function  $F(\theta)$  where  $\theta$  is a  $k$ -dimensional vector and that  $F$  is differentiable with respect to  $\theta$ . Its gradient is defined as:

$$\nabla F(\theta) = \left\langle \frac{\partial F}{\partial \theta_1}(\theta), \frac{\partial F}{\partial \theta_2}(\theta), \dots, \frac{\partial F}{\partial \theta_k}(\theta) \right\rangle \quad (7.58)$$

The gradient has two crucial properties that are exploited in gradient-based optimization. First, the gradient  $\nabla F$  is a vector field that points in the direction

of the greatest increase of  $F$  and whose magnitude indicates the rate of increase. Second, if  $\theta^*$  is a (local) minimum of  $F$ , then the following is true:

$$\nabla F(\theta^*) = 0 \tag{7.59}$$

An extremely simple gradient-based minimization algorithm produces a series of parameter estimates  $\theta^{(1)}, \theta^{(2)}, \dots$  by starting with some initial parameter settings  $\theta^{(1)}$  and updating parameters through successive iterations according to the following rule:

$$\theta^{(i+1)} = \theta^{(i)} - \eta^{(i)} \nabla F(\theta^{(i)}) \tag{7.60}$$

The parameter  $\eta^{(i)} > 0$  is a learning rate which indicates how quickly the algorithm moves along the gradient during iteration  $i$ . Provided this value is small enough that  $F$  decreases, this strategy will find a local minimum of  $F$ . However, while simple, this update strategy may converge slowly, and proper selection of  $\eta$  is non-trivial. More sophisticated algorithms perform updates that are informed by approximations of the second derivative, which are estimated by successive evaluations of  $\nabla F(\theta)$ , and can converge much more rapidly [96].

**Gradient-based optimization in MapReduce.** Gradient-based optimization algorithms can often be implemented effectively in MapReduce. Like EM, where the structure of the model determines the specifics of the realization, the details of the function being optimized determines how it should best be implemented, and not every function optimization problem will be a good fit for MapReduce. Nevertheless, MapReduce implementations of gradient-based optimization tend to have the following characteristics:

- Each optimization iteration is one MapReduce job.
- The objective should decompose linearly across training instances. This implies that the gradient also decomposes linearly, and therefore mappers can process input data in parallel. The values they emit are pairs  $\langle F(\theta), \nabla F(\theta) \rangle$ , which are linear components of the objective and gradient.
- Evaluation of the function and its gradient is often computationally expensive because they require processing lots of data. This makes parallelization with MapReduce worthwhile.
- Whether more than one reducer can run in parallel depends on the specific optimization algorithm being used. Some, like the trivial algorithm of Equation 7.60 treat the dimensions of  $\theta$  independently, whereas many are sensitive to global properties of  $\nabla F(\theta)$ . In the latter case, parallelization across multiple reducers is non-trivial.
- Reducer(s) sum the component objective/gradient pairs, compute the total objective and gradient, run the optimization algorithm, and emit  $\theta^{(i+1)}$ .

- Many optimization algorithms are stateful and must persist their state between optimization iterations. This may either be emitted together with  $\theta^{(i+1)}$  or written to the distributed file system as a side effect of the reducer. Such external side effects must be handled carefully; refer to Section 2.2 for a discussion.

**Parameter learning for log-linear models.** Gradient-based optimization techniques can be quite effectively used to learn the parameters of probabilistic models with a log-linear parameterization [100]. While a comprehensive introduction to these models is beyond the scope of this book, such models are used extensively in text processing applications, and their training using gradient-based optimization, which may otherwise be computationally expensive, can be implemented effectively using MapReduce. We therefore include a brief summary.

Log-linear models are particularly useful for supervised learning (unlike the unsupervised models learned with EM), where an annotation  $\mathbf{y} \in \mathcal{Y}$  is available for every  $\mathbf{x} \in \mathcal{X}$  in the training data. In this case, it is possible to directly model the conditional distribution of label given input:

$$\Pr(\mathbf{y}|\mathbf{x};\theta) = \frac{\exp \sum_i \theta_i \cdot H_i(\mathbf{x}, \mathbf{y})}{\sum_{\mathbf{y}'} \exp \sum_i \theta_i \cdot H_i(\mathbf{x}, \mathbf{y}')} \quad (7.61)$$

In this expression,  $H_i$  are real-valued functions sensitive to features of the input and labeling. The parameters of the model is selected so as to minimize the negative conditional log likelihood of a set of training instances  $\langle \langle \mathbf{x}, \mathbf{y} \rangle_1, \langle \mathbf{x}, \mathbf{y} \rangle_2, \dots \rangle$ , which we assume to be i.i.d.:

$$F(\theta) = \sum_{\langle \mathbf{x}, \mathbf{y} \rangle} -\log \Pr(\mathbf{y}|\mathbf{x};\theta) \quad (7.62)$$

$$\theta^* = \arg \min_{\theta} F(\theta) \quad (7.63)$$

As Equation 7.62 makes clear, the objective decomposes linearly across training instances, meaning it can be optimized quite well in MapReduce. The gradient derivative of  $F$  with respect to  $\theta_i$  can be shown to have the following form [141]:<sup>16</sup>

$$\frac{\partial F}{\partial \theta_i}(\theta) = \sum_{\langle \mathbf{x}, \mathbf{y} \rangle} [H_i(\mathbf{x}, \mathbf{y}) - \mathbb{E}_{\Pr(\mathbf{y}'|\mathbf{x};\theta)}[H_i(\mathbf{x}, \mathbf{y}')] ] \quad (7.64)$$

The expectation in the second part of the gradient's expression can be computed using a variety of techniques. However, as we saw with EM, when very large event spaces are being modeled, as is the case with sequence labeling, enumerating all possible values  $\mathbf{y}$  can become computationally intractable. And, as was the case with HMMs, independence assumptions can be used to enable efficient computation using dynamic programming. In fact, the forward-backward

<sup>16</sup>This assumes that when  $\langle \mathbf{x}, \mathbf{y} \rangle$  is present the model is fully observed (i.e., there are no additional latent variables).

algorithm introduced in Section 7.2 can, with only minimal modification, be used to compute the expectation  $\mathbb{E}_{\text{Pr}(\mathbf{y}'|\mathbf{x};\theta)}[H_i(\mathbf{x}, \mathbf{y}')]$  needed in CRF sequence models, as long as the feature functions respect the same Markov assumption that is made in HMMs. For more information about inference in CRFs using the forward-backward algorithm, we refer the reader to Sha et al. [140].

As we saw in the previous section, MapReduce offers significant speedups when training iterations require running the forward-backward algorithm. The same pattern of results holds when training linear CRFs.

## 7.6 Summary and Additional Readings

This chapter focused on learning the parameters of statistical models from data, using expectation maximization algorithms or gradient-based optimization techniques. We focused especially on EM algorithms for three reasons. First, these algorithms can be expressed naturally in the MapReduce programming model, making them a good example of how to express a commonly-used algorithm in this new framework. Second, many models, such as the widely-used hidden Markov model (HMM) trained using EM, make independence assumptions that permit a high degree of parallelism in both the E- and M-steps. Thus, they are particularly well-positioned to take advantage of large clusters. Finally, EM algorithms are unsupervised learning algorithms, which means that they have access to far more training data than comparable supervised approaches. This is quite important. In Chapter 1, when we hailed large data as the “rising tide that lifts all boats” to yield more effective algorithms, we were mostly referring to unsupervised approaches, given that the manual effort required to generate annotated data remains a bottleneck in many supervised approaches. Data acquisition for unsupervised algorithms is often as simple as crawling specific web sources, given the enormous quantities of data available “for free”. This, combined with the ability of MapReduce to process large datasets in parallel, provides researchers with an effective strategy for developing increasingly-effective applications.

Since EM algorithms are relatively computationally expensive, even for small amounts of data, this led us to consider how related supervised learning models (which typically have much less training data available), can also be implemented in MapReduce. The discussion demonstrates that not only does MapReduce provide a means for coping with ever-increasing amounts of data, but it is also useful for parallelizing expensive computations. Although MapReduce has been designed with mostly data-intensive applications in mind, the ability to leverage clusters of commodity hardware to parallelize computationally-expensive algorithms is an important use case.

**Additional Readings.** Because of its ability to leverage large amounts of training data, machine learning is an attractive problem for MapReduce and an area of active research. Chu et al. [37] presented general formulations of a variety of machine learning problems, focusing on a normal form for ex-

pressing a variety of machine learning algorithms in MapReduce. The Apache Mahout project is an open-source implementation of these and other learning algorithms,<sup>17</sup> and it is also the subject of a forthcoming book [116]. Issues associated with a MapReduce implementation of latent Dirichlet allocation (LDA), which is another important unsupervised learning technique, with certain similarities to EM, have been explored by Wang et al. [151].

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<sup>17</sup> <http://lucene.apache.org/mahout/>