Cloud Computing

Link Analysis in the Cloud

Dell Zhang
Birkbeck, University of London
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Graph Problems & Representations
What is a Graph?

• $G = (V,E)$, where
  – $V$ represents the set of vertices (nodes)
  – $E$ represents the set of edges (links)
  – Both vertices and edges may contain additional information (e.g., edge weights)

• Different types of graphs:
  – directed vs. undirected edges
  – presence or absence of cycles
Source: Wikipedia (Königsberg)
We See Graphs Everywhere

• Ubiquitous network (graph) data
  • Technological Network
    • Internet
  • Information Network
    • WWW, Semantic Web/Ontologies, XML/RD
  – Social network
  – Biological Network
  – Financial Network
  – Transportation Network

Semantic Search, Guha et. al., WWW’03
Some Graph Problems

• Finding shortest paths
  – Routing Internet traffic and UPS trucks

• Finding minimum spanning trees
  – Telecommunication companies laying down fibre

• Finding max flow
  – Airline scheduling
Some Graph Problems

• Identify “special” nodes and communities
  – Breaking up terrorist cells, spread of avian flu
• Bipartite matching
  – Monster.com, Match.com
• And of course... PageRank
Challenge in Dealing with Graph Data

• Flat Files
  – No query support

• RDBMS
  – Can store the graph
  – But limited support for graph query
    • Connect-By (Oracle)
    • Common Table Expressions (CTEs) (Microsoft)
    • Temporal Table
Native Graph Databases

• An Emerging Field

• Storage and Basic Operators
  – **Neo4j** (an open source graph database), InfiniteGraph, VertexDB, ...

• Distributed Graph Processing (mostly in-memory-only)
  – Google’s Pregel, GraphLab, ...
The Graph Analytics Industry

• Status of Practice
  – Graph data in many industries
  – Graph analytics are powerful and can bring great business values/insights
  – Graph analytics not utilized enough in enterprises due to lack of available platforms/tools (except leading tech companies which have high caliber in house engineering teams and resources)
Graphs and MapReduce

• Graph algorithms typically involve:
  – Performing computations at each node: based on node features, edge features, and local link structure
  – Propagating computations: “traversing” the graph

• Key questions:
  – How do you represent graph data in MapReduce?
  – How do you traverse a graph in MapReduce?
Representing Graphs

• Two common representations
  – Adjacency matrix
  – Adjacency list
Adjacency Matrices

• Represent a graph as an $n \times n$ square matrix $M$
  
  - $n = |V|$
  
  - $M_{ij} = 1$ means a link from node $i$ to $j$

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Adjacency Matrices: Critique

• Advantages:
  – Amenable to mathematical manipulation
  – Iteration over rows and columns corresponds to computations on out-links and in-links

• Disadvantages:
  – Lots of zeros for sparse matrices
  – Lots of wasted space
Adjacency Lists

• Take adjacency matrices... and throw away all the zeros

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3
Adjacency Lists: Critique

• Advantages:
  – Much more compact representation
  – Easy to compute over out-links

• Disadvantages:
  – Much more difficult to compute over in-links
Parallel Breadth-First Search

```
        1
      /  \
     2    3
    / \  /  \  \
  5   6 4   7 8
 / \ / \ / \ / \  \
9  10 11 12
```
Single Source Shortest Path

• **Problem**: find shortest path from a source node to one or more target nodes
  – “shortest” might also mean lowest weight or cost
• First, a refresher: Dijkstra’s algorithm
Dijkstra’s Algorithm

Example from CLR
Dijkstra's Algorithm

Example from CLR
Dijkstra’s Algorithm

Example from CLR
Dijkstra’s Algorithm

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Example from CLR
Dijkstra’s Algorithm

Example from CLR
Single Source Shortest Path

- **Problem:** find shortest path from a source node to one or more target nodes
  - “shortest” might also mean lowest weight or cost
- On a single machine: Dijkstra’s algorithm
- MapReduce: Parallel Breadth-First Search (BFS)
  - Consider simplest case of equal edge weights first
  - Solution to the problem can be defined inductively
Finding the Shortest Path

• Here’s the intuition:
  – Define: $b$ is reachable from $a$ if $b$ is in the adjacency list of $a$
  – $\text{DISTANCETo}(s) = 0$
  – For all nodes $p$ reachable from $s$:
    $\text{DISTANCETo}(p) = 1$
  – For all nodes $n$ reachable from some other set of nodes $M$:
    $\text{DISTANCETo}(n) = 1 + \min_{m \in M} \text{DISTANCETo}(m)$
Finding the Shortest Path
Visualizing Parallel BFS
From Intuition to Algorithm

• Data representation:
  – Key:
    node $n$
  – Value:
    $d$ (distance from start),
    adjacency list (list of nodes reachable from $n$)
  – Initialization:
    for all nodes except the start node, $d = \infty$. 
From Intuition to Algorithm

• Mapper:
  – $\forall m \in$ adjacency list: emit $(m, d + 1)$

• Sort/Shuffle
  – Groups distances by reachable nodes

• Reducer:
  – Selects the minimum distance path for each reachable node
  – Additional bookkeeping needed to keep track of the actual path
Multiple Iterations Needed

• Each MapReduce iteration advances the “known frontier” by one hop
  – Subsequent iterations include more and more reachable nodes as frontier expands
  – Multiple iterations are needed to explore entire graph

• Preserving graph structure:
  – Problem: Where did the adjacency list go?
  – Solution: mapper emits \((n, \text{adjacency\ list})\) as well
BFS Pseudo-Code

1: class Mapper
2:   method Map(nid n, node N)
3:       d ← N.Distance
4:       Emit(nid n, N) ▷ Pass along graph structure
5:       for all nodeid m ∈ N.AdjacencyList do
6:           Emit(nid m, d + 1) ▷ Emit distances to reachable nodes

1: class Reducer
2:   method Reduce(nid m, [d₁, d₂, ...])
3:       d_min ← ∞
4:       M ← ∅
5:       for all d ∈ counts [d₁, d₂, ...] do
6:           if IsNode(d) then
7:               M ← d
8:           else if d < d_min then
9:               d_min ← d ▷ Look for shorter distance
10:              M.Distance ← d_min ▷ Update shortest distance
11:             Emit(nid m, node M)
Stopping Criterion

• How many iterations are needed in parallel BFS (equal edge weight case)?

• Convince yourself: when a node is first “discovered”, we’ve found the shortest path

• Now answer the question...
  – Six degrees of separation?

• Practicalities of implementation in MapReduce
Weighted Edges

• Now add positive weights to the edges
  – Why can’t edge weights be negative?

• Simple change: adjacency list now includes a weight $w$ for each edge
  – In mapper, emit $(m, d + w_p)$ instead of $(m, d + 1)$ for each node $m$

• That’s it?
Stopping Criterion

• How many iterations are needed in parallel BFS (positive edge weight case)?
• Convince yourself: when a node is first “discovered”, we’ve found the shortest path

Not true!
Additional Complexities

How many iterations are required to discover the shortest distances to all nodes from $n_1$?
Stopping Criterion

• How many iterations are needed in parallel BFS (positive edge weight case)?
• Practicalities of implementation in MapReduce
Comparison to Dijkstra

• Dijkstra’s algorithm is more efficient
  – At any step it only pursues edges from the minimum-cost path inside the frontier

• MapReduce explores all paths in parallel
  – Lots of “waste”
    – Useful work is only done at the “frontier”

• Why can’t we do better using MapReduce?
Implementation on Hadoop

http://goo.gl/TEoU4
Graphs and MapReduce

• Generic recipe:
  – Represent graphs as adjacency lists
  – Perform local computations in mapper
  – Pass along partial results via out-links, keyed by destination node
  – Perform aggregation in reducer on in-links to a node
  – Iterate until convergence: controlled by external “driver”
  – Don’t forget to pass the graph structure between iterations
PageRank
Random Walks over the Web

• Random surfer model:
  – User starts at a random Web page
  – User randomly clicks on links, surfing from page to page

• PageRank
  – Characterizes the amount of time spent on any given page
  – Mathematically, a probability distribution over pages
Random Walks over the Web

• PageRank captures the notion of page importance
  – Correspondence to human intuition?
  – One of thousands of features used in Web search
  – Note: query-independent
PageRank: Simplified

\[ t_1 \rightarrow x \]
\[ t_2 \rightarrow x \]
\[ t_n \rightarrow x \]

\[ \ldots \]
PageRank: Simplified

• Given page $x$ with in-links $t_1...t_n$, where
  – $C(t)$ is the out-degree of $t$

$$PR(x) = \sum_{i=1}^{n} \frac{PR(t_i)}{C(t_i)}$$
Example: the Web in 1839

Yahoo

Amazon

Microsoft

\[
\begin{array}{ccc}
  & y & a & m \\
  y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 1 \\
m & 0 & 1/2 & 0 \\
\end{array}
\]
Simulating a Random Walk

- Start with the vector \( \mathbf{v} = [1,1,\ldots,1] \) representing the idea that each Web page is given one unit of importance.
- Repeatedly apply the matrix \( M \) to \( \mathbf{v} \), allowing the importance to flow like a random walk.
- Limit exists, but about 50 iterations is sufficient to estimate final distribution.
Example: the Web in 1839

- Equations \( \mathbf{v} = M \mathbf{v} \):
  
  \[
  \begin{align*}
  y &= y/2 + a/2 \\
  a &= y/2 + m \\
  m &= a/2 \\
  y &= 1, 1, 5/4, 9/8, \ldots, 6/5 \\
  a &= 1, 3/2, 1, 11/8, \ldots, 6/5 \\
  m &= 1, 1/2, 3/4, 1/2, \ldots, 3/5
  \end{align*}
  \]
Solving the Equations

• Because there are no constant terms, these 3 equations in 3 unknowns do not have a unique solution.
• Add in the fact that $y + a + m = 3$ to solve.
• In Web-sized examples, we cannot solve by Gaussian elimination, but we need to use the power method (= iterative solution).
Computing PageRank

• Properties of PageRank
  – Can be computed iteratively
  – Effects at each iteration are local
Computing PageRank

• Sketch of algorithm:
  – Start with seed $PR_i$ values
  – Each page distributes its $PR_i$ “credit” to all of its out-links
  – Each page adds up the “credits” from all of its in-links to compute $PR_{i+1}$
  – Iterate until the values converge
Sample PageRank Iterations

Iteration 1
Sample PageRank Iterations

Iteration 2
PageRank in MapReduce

Map

Reduce

\[ n_1 [n_2, n_4] \]
\[ n_2 [n_3, n_5] \]
\[ n_3 [n_4] \]
\[ n_4 [n_5] \]
\[ n_5 [n_1, n_2, n_3] \]
PageRank Pseudo-Code

1: class Mapper
2:   method Map(nid n, node N)
3:     \[ p \leftarrow N.\text{PageRank}/|N.\text{AdjacencyList}| \]
4:     Emit(nid n, N) \triangleright \text{Pass along graph structure}
5:   for all nodeid \( m \in N.\text{AdjacencyList} \) do
6:     Emit(nid m, p) \triangleright \text{Pass PageRank mass to neighbors}

1: class Reducer
2:   method Reduce(nid m, \([p_1, p_2, \ldots]\))
3:     \[ M \leftarrow \emptyset \]
4:   for all \( p \in \text{counts} \ [p_1, p_2, \ldots] \) do
5:     if IsNode(p) then
6:       \[ M \leftarrow p \] \triangleright \text{Recover graph structure}
7:     else
8:       \[ s \leftarrow s + p \] \triangleright \text{Sum incoming PageRank contributions}
9:     \[ M.\text{PageRank} \leftarrow s \]
10:    Emit(nid m, node M)
Real-World Problems

• Some pages are “dead ends” (no out-links).
  – Such a page causes importance to leak out.

• Some other (groups of) pages are spider traps (all out-links are within the group).
  – Eventually spider traps absorb all importance.
Microsoft becomes a dead end

Yahoo

Amazon

Microsoft

\[
\begin{array}{ccc}
 y & a & m \\
 y & 1/2 & 1/2 & 0 \\
a & 1/2 & 0 & 0 \\
m & 0 & 1/2 & 0 \\
\end{array}
\]
Microsoft becomes a dead end

Equations $v = M \cdot v$

$y = y/2 + a/2$

$a = y/2$

$m = a/2$

\[
\begin{array}{cccccc}
y & 1 & 1 & 3/4 & 5/8 & \ldots & 0 \\
a & 1 & 1/2 & 1/2 & 3/8 & \ldots & 0 \\
m & 1 & 1/2 & 1/4 & 1/4 & & 0 \\
\end{array}
\]
Microsoft becomes a spider trap
Microsoft becomes a spider trap

- Equations $v = Mv$:
  
  $y = y/2 + a/2$
  
  $a = y/2$
  
  $m = a/2 + m$

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Google’s Solution

• “Tax” each page a fixed percentage at each iteration.

• Add the same constant to all pages.

• Models a random walk with a fixed probability of going to a random place next.
Example: with 20% Tax

• Equations \( v = 0.8(Mv) + 0.2 \):

\[
\begin{align*}
v &= 0.8(y/2 + a/2) + 0.2 \\
a &= 0.8(y/2) + 0.2 \\
m &= 0.8(a/2 + m) + 0.2
\end{align*}
\]

\[
\begin{array}{cccccc}
y & 1 & 1.00 & 0.84 & 0.776 & 7/11 \\
a & = & 1 & 0.60 & 0.60 & 0.536 & \ldots & 5/11 \\
m & 1 & 1.40 & 1.56 & 1.688 & 21/11
\end{array}
\]
PageRank: Complete

• Two additional complexities
  – What is the proper treatment of dangling nodes (i.e., nodes with no out-links)?
  – How do we factor in the random jump factor?
PageRank: Complete

• Solution:
  – Second pass to redistribute “missing PageRank mass” and account for random jumps

\[
p' = \alpha \left( \frac{1}{N} \right) + (1 - \alpha) \left( \frac{m}{N} + p \right)
\]

• \( p \) is PageRank value from before,
  \( p' \) is updated PageRank value

• \( N \) is the total number of nodes in the graph

• \( m \) is the missing PageRank mass
PageRank Convergence

• Alternative convergence criteria
  – Iterate until PageRank values don’t change
  – Iterate until PageRank rankings don’t change
  – Fixed number of iterations

• Convergence for web graphs?
Beyond PageRank

• Link structure is important for web search
  – PageRank is one of many link analysis algorithms: HITS, SALSA, etc.
  – Used with thousands of other features in ranking...

• Adversarial nature of web search
  – Link spamming
  – Spider traps
  – Keyword stuffing
  – ...

Efficient Graph Algorithms

• Sparse vs. Dense Graphs
• Graph Topologies
Power Laws are everywhere!

Local Aggregation

• Use combiners!
  – In-mapper combining design pattern also applicable

• Maximize opportunities for local aggregation
  – Simple tricks: sorting the dataset in specific ways
Take Home Messages

• Graph Problems and Representations
• Parallel Breadth-First Search
• PageRank: Simplified and Complete