1. Using logical equivalences prove that the following formulas are equivalent:

(a) \((A \rightarrow B) \land (A \rightarrow C)\) and \(A \rightarrow (B \land C)\);
(b) \(A \rightarrow (B \rightarrow C)\) and \((A \land B) \rightarrow C\).

2. Using logical equivalences show that the following formulas are tautologies (that is, equivalent to 1):

(a) \(((A \land B) \land \neg A) \rightarrow B\);
(b) \(((A \rightarrow B) \rightarrow A) \rightarrow A\).

3. Construct a DFA which accepts those words in the alphabet \(\{a, b\}\) that have an even number of \(a\)'s and an odd number of \(b\)'s.

4. Construct regular expressions for the following languages:

(a) the words from \(\{a, b\}\)^* which contain a subword \(ab\);
(b) the words from \(\{a, b\}\)^* which do not contain a subword \(ab\);
(c) the words from \(\{a, b, c\}\)^* in which \(a\) is always followed by \(c\).
(d) the words from \(\{a, b\}\)^* in which the number of \(a\)'s is even.
(e) the words from \(\{a, b\}\)^* which do not have two \(b\)'s in a row.
(f) the words from \(\{a, b\}\)^* of length 3 and 4.

5. Convert the following NFA into a DFA:

What language does it recognise? Construct a regular expression for this language.