# Fundamentals of Computing

## Tutorial 1: binary representations of numbers

## Model answers

1. Find the decimal representation of the binary numbers $10110011_2$, $0.1011_2$, and $11.001_2$.

   **Answer.** Recall from the lecture notes that a binary number

   $$a_n a_{n-1} \dots a_1 a_0 \,.\, a_{-1} \dots a_{-m}$$

   where the $a_i$ are bits (0 or 1), means the number

   $$a_n \times 2^n + a_{n-1} \times 2^{n-1} + \dots + a_1 \times 2^1 + a_0 \times 2^0 + a_{-1} \times 2^{-1} + \dots + a_{-m} \times 2^{-m}$$

   So $10110011_2 = 2^7 + 2^5 + 2^4 + 2^1 + 2^0 = 128 + 32 + 16 + 2 + 1 = 179_{10}$,
   $0.1011_2 = 2^{-1} + 2^{-3} + 2^{-4} = 0.6875_{10}$, and $11.001_2 = 2^1 + 2^0 + 2^{-3} = 2 + 1 + 0.125 = 3.125_{10}$

   (Try to remember the powers of 2!)

2. Find the binary representation of the decimal numbers $203_{10}$, $0.8125_{10}$, and $203.8125_{10}$.

   **Answer.** To find the binary representation of $203_{10}$, we use the algorithm explained in the lecture slides (FoC-I, page 5): divide by 2 and keep track of the remainders:

   | | |
   |---|---|
   | $203 = 2 \times 101 +$ | 1 |
   | $101 = 2 \times 50 +$ | 1 |
   | $50 = 2 \times 25 +$ | 0 |
   | $25 = 2 \times 12 +$ | 1 |
   | $12 = 2 \times 6 +$ | 0 |
   | $6 = 2 \times 3 +$ | 0 |
   | $3 = 2 \times 1 +$ | 1 |
   | $1 = 2 \times 0 +$ | 1 |

   Now we read the remainders bottom-up and obtain the answer: $11001011_2$.

   As explained on page 15 of FoC-I, the binary representation of $0.8125_{10}$ is computed by multiplying by 2 and keeping track of the resulting integer and fractional parts:

   | | |
   |---|---|
   | $0.8125 \times 2 = 1.625 = 0.625 +$ | integer 1 |
   | $0.625 \times 2 = 1.25 = 0.25 +$ | integer 1 |
   | $0.25 \times 2 = 0.50 = 0.5 +$ | integer 0 |
   | $0.5 \times 2 = 1.00 = 0.0 +$ | integer 1 |

   Thus, $0.8125_{10} = 0.1101_2$. Finally, $203.8125_{10} = 203_{10} + 0.8125_{10} = 11001011.1101_2$.

3. Represent the decimal number $-44_{10}$ as

   (a) a sign-magnitude 32-bit binary number;

   (b) a ones' complement 32-bit binary number;

   (c) a two's complement 32-bit binary number.

   **Answer.** Using the algorithm from the lecture slides (divide by 2 and keep track of the remainders), we obtain

   $$44_{10} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0010\ 1100_2.$$

   (a) The number is negative, so we just flip the left-most bit to 1, which gives the answer

$$1000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0010\ 1100.$$

(b) As explained on page 9 of FoC-I, since the number is negative, we just invert all of the bits in the binary representation of 44 and obtain the answer:

$$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1101\ 0011.$$

(c) As explained on page 12 of FoC-I, we can take the Boolean negation (that is, invert all of the bits) of the binary representation of $44_{10}$

$$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1101\ 0011,$$

and then add 1 obtaining the final answer:

$$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1101\ 0100.$$

4. Given the machine 32-bit word

$$1111\ 1111\ 1111\ 1111\ 1111\ 1101\ 1011\ 1101,$$

find the decimal number represented by this word assuming that it is

(a) a two's complement integer;

(b) an unsigned integer;

(c) a ones' complement integer;

(d) a sign-magnitude integer.

**Answer.** (a) The first formula on page 12 of FoC-I gives the answer:

$$-2^{31} + 2^{30} + \cdots + 2^{10} + 2^8 + 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0$$

which is acceptable. The following argument gives a better answer. The given number $N$ is negative. We can compute its negation $-N$ but inverting the bits and then adding 1, which gives

$$0000\ 0000\ 0000\ 0000\ 0000\ 0010\ 0100\ 0011$$

representing the number $-N = 2^9 + 2^6 + 2^1 + 2^0 = 512 + 64 + 3 = 579_{10}$. So $N = -579_{10}$.

(b) $2^{31} + 2^{30} + \cdots + 2^{10} + 2^8 + 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0$ (= 4 294 966 717). You do not have to compute the sum.

(c) The given ones' complement integer $N$ is negative. We can compute its negation $-N$ but inverting the bits, which gives

$$0000\ 0000\ 0000\ 0000\ 0000\ 0010\ 0100\ 0010$$

representing the number $-N = 2^9 + 2^6 + 2^1 = 512 + 64 + 2 = 578_{10}$. So $N = -578_{10}$.

(d) The first bit represents the sign, so we know that it is a negative number. The rest of the bits represent the magnitude:

$$2^{30} + \cdots + 2^{10} + 2^8 + 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0 = 2\ 147\ 483\ 069$$

So the number is $-2\ 147\ 483\ 069_{10}$.

5. Given the 32-bit word

$$1100\ 0001\ 0101\ 1101\ 0000\ 0000\ 0000\ 0000,$$

find the decimal number represented by this word assuming that it is

(a) a single precision IEEE 754 floating-point number;

(b) a two's complement integer (you can give the number as a sum of powers of 2).

**Answer.** (a) In this case, the word

$$1 \mid 100\ 0001\ 0 \mid 101\ 1101\ 0000\ 0000\ 0000\ 0000$$

gives the sign $S = 1$, the biased exponent $Exp_b = 1000\ 0010_2 = 2^7 + 2^1 = 128 + 2 = 130$, and the fraction $F = 0.1011101_2$. Recalling that $Bias = 127$, we obtain

$$(-1)^S \cdot 2^{Exp_b - Bias} \cdot (1 + F) = -1 \cdot 2^3 \cdot 1.1011101_2 = -1101.1101 = -13\frac{13}{16} = -13,8125.$$

(b) Let $N$ be the given number. Then we either represent $N$ as

$$2^{16} + 2^{18} + 2^{19} + 2^{20} + 2^{22} + 2^{24} + 2^{30} - 2^{31}$$

or we first find the absolute value of the number by inverting all bits and adding 1, which gives

$$-N = 2^{29} + 2^{28} + 2^{27} + 2^{26} + 2^{25} + 2^{23} + 2^{21} + 2^{17} + 2^{16}$$

So

$$N = -2^{29} - 2^{28} - 2^{27} - 2^{26} - 2^{25} - 2^{23} - 2^{21} - 2^{17} - 2^{16}.$$

6. Represent 16.6875 as a single precision IEEE 754 floating-point number.

**Answer.** First, we convert the number to the binary normalised scientific notation:

$$16.6875_{10} = 16\frac{11}{16} = 10000.1011_2 = 1.00001011 \times 2^4$$

as explained on page 15 of FoC-I. Thus, $S = 0$, $E = 4$, $Exp_b = E + Bias = 131_{10} = 10000011_2$, $F = 0.00001011_2$, and so the answer is

$$0100\ 0001\ 1000\ 0101\ 1000\ 0000\ 0000\ 0000.$$