RDF semantics

RDF semantics is given by the RDF **Model Theory**;

see http://www.w3.org/TR/rdf-mt/

Model theory assumes that the language refers to a **'world'**, and describes the minimal conditions that a world must satisfy in order to assign an appropriate meaning for every expression in the language.

A particular world is called an **interpretation**, so that model theory might be better called **'interpretation theory'**. The idea is to provide an abstract, mathematical account of the properties that any such interpretation must have, making as few assumptions as possible about its actual nature or intrinsic structure, thereby retaining as much generality as possible.

> The chief utility of a formal semantic theory is not to provide any deep analysis of the nature of the things being described by the language or to suggest any particular processing model, but rather to provide a technical way to determine when inference processes are valid, i.e., when they preserve **truth**.

A bit of ‘maths’: Sets

A **set** is a collection of objects, called **elements** or **members**

- The set, call it A, whose elements are *Shakespeare, Tasmania, and Monday*
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Notation

$A = \{\textit{Shakespeare}, \textit{Tasmania}, \textit{Monday}\}$ list all elements and enclose with $\{\ \}$

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Write $a \in S$ to say that a is an element of S (from Greek ε)
Write $a \notin S$ to say that a is not an element of S

$8 \in B$ $7 \notin B$ $C = \{x \mid x \in \mathbb{N} \text{ and } x \notin B\}$

where $\mathbb{N}$ is the set of all natural numbers. What is C?

Fundamental features of sets

- **A set must be distinguished from its description**

  For instance, the following descriptions define the same set:

$$\{2,3,4\} \quad \{3,2,4\} \quad \{2,2,3,4,4\} \quad \{x \in \mathbb{N} \mid 2 \leq x \leq 4\} \quad \{y \in \mathbb{N} \mid 1 < y < 5\}$$

- **All elements of a set are distinct**

  In other words, no element may ‘occur’ more than once in a set

  We do not distinguish between $\{3,2,4\}$ and $\{2,2,3,4,4\}$

- **The elements of a set are not ordered in any way**

  We do not distinguish between $\{3,2,4\}$ and $\{2,3,4\}$

- A set can be an element of another set

  For example, $\{0,\{0\}\}$ has two elements: 0 and $\{0\}$

- A set can be **empty**, that is without elements. This set is denoted by $\emptyset$

Subsets

A set B is a **subset** of a set A if every element of B is an element of A

Notation: $B \subseteq A$. Also say: B is **included** in A

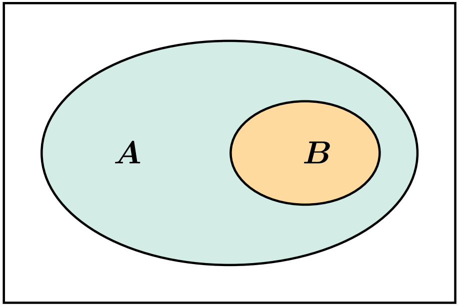

Figure 1: **Venn diagram** of $B \subseteq A$.

John Venn was a 19th-century British philosopher and mathematician who introduced the Venn diagram in 1881

$\{3, 4, 5\} \subseteq \{1, 5, 4, 2, 1, 3\}$ $\{3, 3, 5\} \subseteq \{3, 5\}$ $\{5, 3\} \subseteq \{3, 5\}$

$\emptyset \subseteq A$ for any set A.

What is 'iff' ?

If and only if (or **iff**) is a biconditional logical connective between statements:
it is the standard conditional **if** combined with its reverse **only if**.
The truth of either one of the connected statements requires the truth of the other,
i.e., either both statements are true, or both are false.

Alternative phrases to P 'iff Q':

- P is necessary and sufficient for Q
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Powersets

The **powerset** of a set A is defined to be the set of **all** subsets of A

Notation: $\textbf{Pow}(A) = \{X \mid X \subseteq A\}$

Examples:

1. Let $A = \{2\}$. Then
$$\textbf{Pow}(A) = \{\emptyset, \{2\}\}$$
2. Let $B = \{1, 2, 3\}$. Then
$$\textbf{Pow}(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$$
3. Let $C = \emptyset$. Then
$$\textbf{Pow}(C) = \{\emptyset\} \qquad (\neq \emptyset)$$

More common notation: $\textbf{Pow}(A) = 2^A$

Why?

Because when A has n elements, then $\textbf{Pow}(A)$ has 2^n elements

Sequences, tuples, and Cartesian products

A **sequence** of objects is a list of these objects **taken in a certain order**

The sequences $(1, 2, 3)$ $(2, 1, 3)$ $(3, 1, 2)$ are **different**

The sets $\{1, 2, 3\}$ $\{2, 1, 3\}$ $\{3, 1, 2\}$ are **the same**

Finite sequences are called **tuples**. A sequence with k elements is a **k-tuple**

A 2-tuple is also called a **pair**

The **Cartesian product** $A \times B$ of sets A and B is the *set* of all pairs (a, b) where $a \in A$ and $b \in B$

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Example. Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Then

$$A \times B = \{(1, a), (2, a), (1, b), (2, b), (1, c), (2, c)\}$$

RDF semantics (cont.)

- Let V be a vocabulary containing all names (URIs and literals) occurring in RDF triples
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An RDF triple $\langle s, p, o \rangle$ is **true** in I if and only if
$s, p, o \in V$, $IS(p) \in IP$ and $(IS(s), IS(o)) \in IEXT(IS(p))$

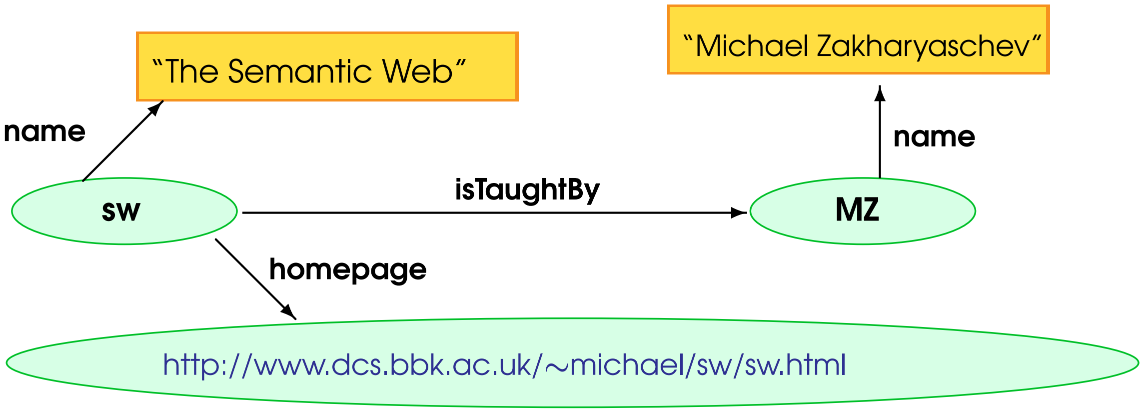
An RDF triple is false in I if it is not true in I

An RDF graph is **true** in I if and only if every triple of it is true in I

In particular, it is false in I if some triple is not true in I

RDF semantics: example

Construct an interpretation in which the following RDF graph is true:



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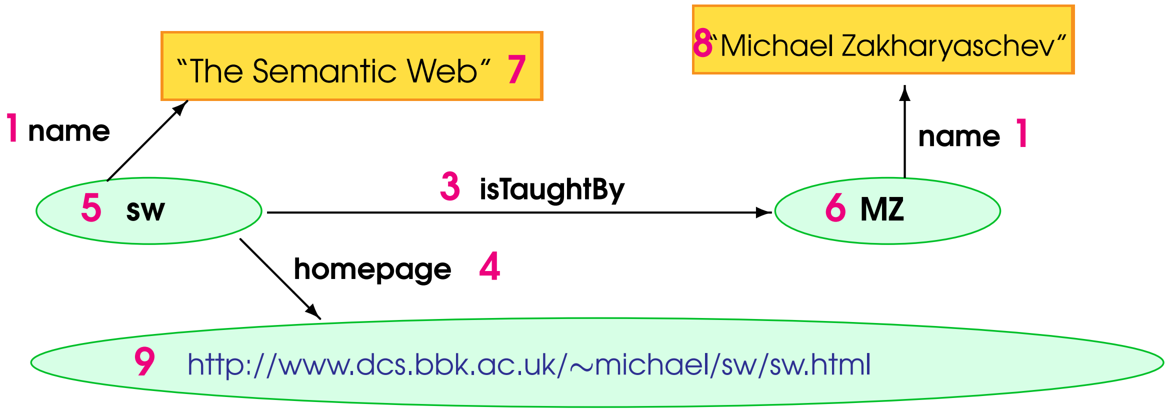

$\mathit{IR} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $\mathit{IP} = \{1, 2, 3, 4\}$

$\mathit{IS}($**name**$) = 1$, $\mathit{IS}($**isTaughtBy**$) = 3$, $\mathit{IS}($**homepage**$) = 4$, $\mathit{IS}($**sw**$) = 5$,
$\mathit{IS}($**MZ**$) = 6$, $\mathit{IS}($http://www.dcs.bbk.ac.uk/∼michael/sw/sw.html$) = 9$,
$\mathit{IS}($"The Semantic Web"$) = 7$, $\mathit{IS}($"Michael Zakharyaschev"$) = 8$

$\mathit{IEXT}(1) = \{(5, 7), (6, 8)\}$, $\mathit{IEXT}(2) = \emptyset$, $\mathit{IEXT}(3) = \{(5, 6)\}$,
$\mathit{IEXT}(4) = \{(5, 9)\}$

RDF semantics (cont.)

- RDF imposes **semantic conditions** on interpretation of RDF symbols (rdf:...), e.g.,
  - $p \in IP$ if and only if $(p, IS(\text{rdf:Property})) \in IEXT(IS(\text{rdf:type}))$

(IP is the set of resources that have the value IS(rdf:Property) of the property IS(rdf:type))

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- Also, certain **axiomatic triples** must be true in every RDF interpretation, e.g.,
  - ⟨rdf:type, rdf:type, rdf:Property⟩ (‘type is of type property’)
  - ⟨rdf:subject, rdf:type, rdf:Property⟩
  - ⟨rdf:predicate, rdf:type, rdf:Property⟩
  - ⟨rdf:object, rdf:type, rdf:Property⟩
  - ⟨rdf:first, rdf:type, rdf:Property⟩
  - ⟨rdf:rest, rdf:type, rdf:Property⟩
  - ...

RDFS Semantics

RDF Schema introduces schema vocabulary (e.g., **rdfs:subClassOf**, **rdfs:Class**) in RDF

RDFS graphs are interpreted in structures that are similar to RDF interpretations.

It simply adds extra semantic conditions that give **meaning** to the **schema vocabulary**

Remember: everything in RDF/RDFS is a resource, even a property and a class