Tableau algorithm: examples

For each of the following concepts, say if it is satisfiable or not and show how the tableaux algorithm would use a sequence of completion rules to prove the (un)satisfiability.

(1) $A \sqcap \exists R.C \sqcap \forall R.D$

(2) $\exists R.C \sqcap \forall R.\neg(C \sqcap D)$

(3) $A \sqcap \exists R.C \sqcap \forall R.D \sqcap \forall R.\neg(C \sqcap D)$

(4) $\exists R.(A \sqcap \exists R.C) \sqcap \forall R.\neg C$

(5) $\exists R.(A \sqcap \exists R.C) \sqcap \forall R.\forall R.\neg C$

(6) $\neg C \sqcap \exists R.C \sqcap \forall R.(\neg C \sqcup \exists R.C)$

(7) $A \sqcap \forall R.A \sqcap \forall R.\neg\exists P.A \sqcap \exists R.\exists P.A$

Tableau algorithm: example 6

S_0	$= \{\ x\colon \neg C \sqcap \exists R.C \sqcap \forall R.(\neg C \sqcup \exists R.C)\ \}$
$S_0 \rightarrow_{\sqcap} S_1$	$= S_0 \cup \{\ x\colon \neg C,\ x\colon \exists R.C,\ x\colon \forall R.(\neg C \sqcup \exists R.C)\ \}$
$S_1 \rightarrow_{\exists} S_2$	$= S_1 \cup \{\ (x,y)\colon R,\ y\colon C\ \}$
$S_2 \rightarrow_{\forall} S_3$	$= S_2 \cup \{\ y\colon\ \neg C \sqcup \exists R.C\ \}$
$+\ S_3 \rightarrow_{\sqcup} S_{4.1}$	$= S_3 \cup \{\ y\colon \neg C\ \}$ — clash
$+\ S_3 \rightarrow_{\sqcup} S_{4.2}$	$= S_3 \cup \{\ y\colon \exists R.C\ \}$
$S_{4.2} \rightarrow_{\exists} S_{5.2}$	$= S_{4.2} \cup \{\ (y,z)\colon R,\ z\colon C\ \}$ — complete and clash-free

The concept is satisfiable in the interpretation $\mathcal{I}_6 = \langle \Delta^{\mathcal{I}_6}, \cdot^{\mathcal{I}_6} \rangle$, where

$$\Delta^{\mathcal{I}_6} = \{x, y, z\}, \quad C^{\mathcal{I}_6} = \{y, z\} \quad \text{and} \quad R^{\mathcal{I}_6} = \{(x,y), (y,z)\}$$

Tableau algorithm: example 7 ('mad cows')

First, transform into NNF. Then

S_0	$= \{\ x\colon A \sqcap \forall R.A \sqcap \forall R.\forall P.\neg A \sqcap \exists R.\exists P.A\ \}$
$S_0 \to_\sqcap S_1$	$= S_0 \cup \{\ x\colon A,\ x\colon \forall R.A,\ x\colon \forall R.\forall P.\neg A,\ x\colon \exists R.\exists P.A\}$
$S_1 \to_\exists S_2$	$= S_1 \cup \{\ (x, y)\colon R,\ y\colon \exists P.A\ \}$
$S_2 \to_\forall S_3$	$= S_2 \cup \{\ y\colon A\ \}$
$S_3 \to_\forall S_4$	$= S_3 \cup \{\ y\colon \forall P.\neg A\ \}$
$S_4 \to_\exists S_5$	$= S_4 \cup \{\ (y, z)\colon P,\ z\colon A\ \}$
$S_5 \to_\forall S_6$	$= S_5 \cup \{\ z\colon \neg A\ \}$ — clash

The concept is not satisfiable since all branches of the tableau contain clashes

Mad cows example reading: A stands for Animal, R for eats, P for isPartOf
$\forall R.A \sqcap \forall R.\neg\exists P.A$ says 'cows are vegetarians' (and so should be mad cows) and
$\exists R.\exists P.A$ says 'mad cows eat sheep brain'