Exercises, tutorial on 28 November 2011

1. (a) Consider the interpretation $\mathcal{I} = (\Delta, \cdot^{\mathcal{I}})$, where

$$\Delta^{\mathcal{I}} = \{a, b, c, d, e, f\} \quad A^{\mathcal{I}} = \{a, b, f\}, \quad B^{\mathcal{I}} = \{b, d, f\},$$

$$R^{\mathcal{I}} = \{(a, b), (a, c), (b, c), (c, e), (c, d)\}.$$

   Represent this interpretation as a labelled graph.
   Compute $(\neg B \sqcup \neg A)^{\mathcal{I}}$, $(\exists R.(\neg B \sqcup \neg A))^{\mathcal{I}}$, $(\forall R.(B \sqcap A))^{\mathcal{I}}$, $(\forall R.\forall R(B \sqcap A))^{\mathcal{I}}$.

   (b) Consider the interpretation $\mathcal{I} = (\Delta, \cdot^{\mathcal{I}})$, where

$$\Delta^{\mathcal{I}} = \{1, 2, 3, 4, 5\} \quad A^{\mathcal{I}} = \{2, 4\}, \quad B^{\mathcal{I}} = \{1, 2, 3\},$$

$$R^{\mathcal{I}} = \{(2, 3), (2, 1), (5, 1), (4, 5), (3, 2), (3, 3)\}.$$

   Represent this interpretation as a labelled graph.
   Compute $(A \sqcup \exists R.B)^{\mathcal{I}}$, $(B \sqcap \forall R.\neg A)^{\mathcal{I}}$, $(\forall R.((B \sqcap \forall R.\neg A) \sqcup \neg B))^{\mathcal{I}}$.

2. Represent the following concepts in negation normal form (NNF):

   (a) $\neg(\exists R.(\neg A \sqcup B) \sqcap \forall S.((A \sqcap \neg B) \sqcup \exists R.A))$

   (b) $\neg(\forall S.((C \sqcap \exists R.\forall S.A) \sqcap \forall S.(A \sqcap \exists R.\exists R.A)))$

   (c) $\neg(B \sqcap (\exists R.C \sqcup \forall S.\forall R.(\exists R.A \sqcap \exists S.\neg A)))$.

3. Using the tableau algorithm, determine whether the following concepts are satisfiable:

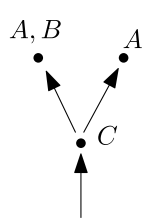
   (a) $\neg(\forall R.A \sqcup \exists R(\neg A \sqcap \neg B))$

   (b) $\neg\forall R.\forall R.P \sqcap \forall R.P$

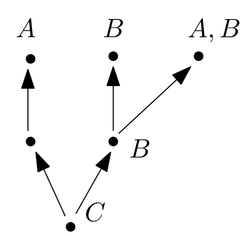
   (c) $\exists R.(\forall S.C) \sqcap \forall R.(\exists S.\neg C)$

   (d) $(\exists S.C \sqcap \exists S.D) \sqcap \forall S.(\neg C \sqcup \neg D)$

4. Give examples of concepts for which the tableau algorithm generates the following interpretations:



$a)$



$b)$

5. Use the tableau algorithm to check whether the following concept inclusions always hold in every interpretation:

   (a) $\forall R.(\neg A \sqcup B) \sqsubseteq \neg\forall R.A \sqcup \forall R.B$

   (b) $\neg\forall R.A \sqcup \forall R.B \sqsubseteq \forall R.(\neg A \sqcup B)$

   (c) $\neg\forall R.A \sqcap \forall R((\forall R.B) \sqcup A)) \sqsubseteq \forall R.\neg(\exists R.A) \sqcap \exists R.(\exists R.B)$