**Solutions to the exercises**

1. Consider the TBox $\mathcal{T}$ with the following axioms:

$$\begin{aligned} \forall R.\neg B &\sqsubseteq B \\ \exists R.(\exists R.C) &\sqsubseteq \neg A \sqcup \neg B \end{aligned}$$

and the interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where

$$\begin{aligned} \Delta^{\mathcal{I}} &= \{a, b, c, d, e, f\} \\ A^{\mathcal{I}} &= \{a, c, e\} \\ B^{\mathcal{I}} &= \{c, d, e, f\} \\ C^{\mathcal{I}} &= \{e\} \\ R^{\mathcal{I}} &= \{(a, f), (a, c), (b, d), (d, c), (c, e), (f, a)\} \end{aligned}$$

Is $\mathcal{I}$ a model of $\mathcal{T}$?

**Solution**. We have to compute $(\forall R.\neg B)^{\mathcal{I}}$, $(\exists R.(\exists R.C))^{\mathcal{I}}$ and $(\neg A \sqcup \neg B)^{\mathcal{I}}$:

$$\begin{aligned} (\neg B)^{\mathcal{I}} &= \{a, b, c, d, e, f\} \setminus \{c, d, e, f\} = \{a, b\} \\ (\forall R.\neg B)^{\mathcal{I}} &= \{e, f\} \\ A^{\mathcal{I}} &= \{a, c, e\} \\ (\exists R.C)^{\mathcal{I}} &= \{c\} \\ (\exists R.(\exists R.C))^{\mathcal{I}} &= \{a, d\} \\ (\neg A)^{\mathcal{I}} &= \{d, e, f\} \\ (\neg A \sqcup \neg B)^{\mathcal{I}} &= \{a, b, d, e, f\} \end{aligned}$$

So, wee see that $(\forall R.\neg B)^{\mathcal{I}} \subseteq B^{\mathcal{I}}$ and $(\exists R.(\exists R.C))^{\mathcal{I}} \subseteq (\neg A \sqcup \neg B)^{\mathcal{I}}$. Therefore, $\mathcal{I}$ is a model of $\mathcal{T}$.

2. Consider the ABox $\mathcal{A}$:

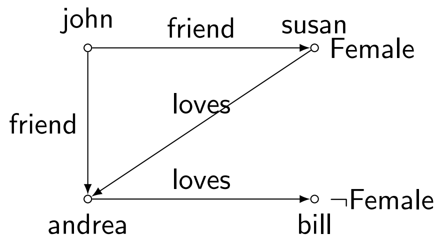
1. $(\mathsf{john}, \mathsf{susan}) : \mathsf{friend}$
2. $(\mathsf{john}, \mathsf{andrea}) : \mathsf{friend}$
3. $(\mathsf{susan}, \mathsf{andrea}) : \mathsf{loves}$
4. $(\mathsf{andrea}, \mathsf{bill}) : \mathsf{loves}$
5. $\mathsf{susan} : \mathsf{Female}$
6. $\mathsf{bill} : \neg\mathsf{Female}$

Represent the following query (to $\mathcal{A}$) as an inference service problem and find an answer:

"Does John have a female friend who is in love with a male (not female) person?"

**Solution.** The query can be represented as the instance checking problem: is john an instance of the concept $\exists$friend.(Female $\sqcap$ $\exists$loves.$\neg$Female) w.r.t. the ABox $\mathcal{A}$? This means that we have to check whether john is an instance of $\exists$friend.(Female $\sqcap$ $\exists$loves.$\neg$Female) in every interpretation $\mathcal{I}$ satisfying $\mathcal{A}$.

So, suppose that $\mathcal{I}$ is an *arbitrary model* of $\mathcal{A}$. This means that the following hold in $\mathcal{I}$:




But we don't know what else holds in $\mathcal{I}$. Anyway, we have to find out whether john belongs to $\exists$friend.(Female $\sqcap$ $\exists$loves.$\neg$Female) in $\mathcal{I}$. Although we don't know whether andrea is a female or not, we can consider two possible cases:

*Case* 1: andrea belongs to Female in $\mathcal{I}$. Then john has a female friend, namely andrea, who is in love with a male, namely bill.

*Case* 2: andrea doesn't belong to Female in $\mathcal{I}$. Then john has a female friend, namely susan, who is in love with a male, namely andrea.

In either case, john belongs to $\exists$friend.(Female $\sqcap$ $\exists$loves.$\neg$Female) in $\mathcal{I}$.

2. Using the individuals john and cnets, concepts Course, Lecturer, MSc and BSc, and roles teaches and hasDegree, represent the following knowledge base as an $\mathcal{ALC}$ knowledge base $\mathcal{K}$:

- Everybody who teaches a course must either have an MSc degree or be a lecturer.
- Every lecturer teaches some course.
- Every lecturer has a BSc degree.
- Everybody with an MSc degree has a BSc degree as well.
- John teaches the Computer Networks course.

(i) Is the statement "John has an MSc degree" a logical consequence of the knowledge base $\mathcal{K}$? Explain your answer.

(ii) Is the statement "Everybody who teaches a course must have a BSc degree" a logical consequence of the knowledge base $\mathcal{K}$? Explain your answer.

**Solution.** The knowledge base $\mathcal{K}$ consists of the following TBox axioms:

- $\exists$teaches.Course $\sqsubseteq$ $\exists$hasDegree.MSc $\sqcup$ Lecturer
- Lecturer $\sqsubseteq$ $\exists$teaches.Course
- Lecturer $\sqsubseteq$ $\exists$hasDegree.BSc
- $\exists$hasDegree.MSc $\sqsubseteq$ $\exists$hasDegree.BSc

and two ABox assertions: $(\mathsf{john}, \mathsf{cnets}) : \mathsf{teaches}$ and $\mathsf{cnets} : \mathsf{Course}$.

(i) The statement "John has an MSc degree" is not a logical consequence of the above knowledge base since there is a model $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ of $\mathcal{K}$ such that $\mathsf{john} : \exists \mathsf{hasDegree}.\mathsf{MSc}$ is not satisfied in it:

$$
\begin{aligned}
\Delta^{\mathcal{I}} &= \{j, b, c\}, & & \\
\mathsf{Course}^{\mathcal{I}} &= \{c\}, & \mathsf{Lecturer}^{\mathcal{I}} &= \{j\}, \\
\mathsf{MSc}^{\mathcal{I}} &= \emptyset, & \mathsf{BSc}^{\mathcal{I}} &= \{b\}, \\
\mathsf{teaches}^{\mathcal{I}} &= \{(j, c)\}, & \mathsf{hasDegree}^{\mathcal{I}} &= \{(j, b)\}, \\
\mathsf{john}^{\mathcal{I}} &= j & \mathsf{cnets}^{\mathcal{I}} &= c.
\end{aligned}
$$

i.e., John is a lecturer but has only a BSc degree.

(ii) The statement "Everybody who teaches a course must have a BSc degree" is a logical consequence of $\mathcal{K}$ since

- by the first axiom, everybody who teaches a course must either have an MSc degree or be a lecturer; we consider two cases:
  - (has an MSc degree) then, by the fourth axiom, everybody with an MSc degree has a BSc degree as well;
  - (is a lecturer) then, by the third axiom, lecturer has a BSc degree;
- therefore, everybody who teaches a course has a BSc degree.