Tractable Interval Temporal Propositional and Description Logics

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Abstract

We design a tractable Horn fragment of the Halpern-Shoham temporal logic and extend it to interval-based temporal description logics, instance checking in which is P-complete for both combined and data complexity.

Introduction

The aims of this paper are to (*i*) design a tractable sub-Boolean fragment of the Halpern-Shoham interval temporal logic $\mathcal{HS}$ (Halpern and Shoham 1991) and (*ii*) construct on its basis tractable descriptions logics with temporal interval operators. The design of these logics is motivated by possible applications in ontology-based data access over temporal databases (which will be discussed at the end of the paper).

The Halpern-Shoham logic $\mathcal{HS}$ is an extension of propositional logic with temporal operators of the form $\langle \mathsf{R} \rangle$, where R is one of Allen's (1981) interval relations (*after*, *begins*, *ends*, *during*, *later*, *overlaps*, *equals* and their inverses). The propositional variables of $\mathcal{HS}$ are interpreted by sets of closed intervals $[i, j]$ of some flow of time (such as $\mathbb{Z}$, $\mathbb{R}$, etc.), and a formula $\langle \mathsf{R} \rangle \varphi$ is regarded to be true in $[i, j]$ iff φ is true in some interval $[i', j']$ such that $[i, j]\mathsf{R}[i', j']$ in Allen's interval algebra. Unfortunately, this natural and seemingly simple logic turned out to be highly undecidable (Halpern and Shoham 1991). One explanation of the bad computational behaviour of $\mathcal{HS}$ is that it can be viewed as a two-dimensional modal logic interpreted over products of (linear) Kripke frames, which provide a good playground for simulating Turing machines, tilings, lossy channels, etc.; see, e.g., (Marx and Venema 1997; Marx and Reynolds 1999; Reynolds and Zakharyaschev 2001; Gabelaia et al. 2005; Konev, Wolter, and Zakharyaschev 2005; Gabelaia et al. 2006; Hampson and Kurucz 2014).

The interest in interval temporal logics was renewed in the 2000s when decidable fragments of $\mathcal{HS}$ were constructed by restricting the available sets of temporal operators (Bresolin et al. 2009). The reader can check the current decidability status of numerous fragments of $\mathcal{HS}$ over various timelines at `itl.dimi.uniud.it/content/logic-hs`; see also (Lodaya 2000; Bresolin et al. 2012b; 2012a; 2014a; Marcinkowski and Michaliszyn 2014; Bresolin et al. 2014b; Montanari, Puppis, and Sala 2014) and references therein. The computational complexity of the decidable fragments ranges from NP to EXPSPACE over strongly discrete linear orders, and from NP to non-primitive recursive over finite linear orders. Bresolin, Muñoz-Velasco, and Sciavicco (2014) defined a 'Horn fragment' of $\mathcal{HS}$; alas, over $\mathbb{Z}$ and other strongly discrete linear orders, it turned out to be undecidable.

In this paper, we consider a somewhat different Horn fragment, denoted $\mathcal{HS}_{horn}$, which comprises formulas φ given by the following grammar:

$$\lambda \ ::= \ p \ \mid \ \langle \mathsf{R} \rangle \lambda \ \mid \ [\mathsf{R}]\lambda, \quad \lambda^+ \ ::= \ p \ \mid \ [\mathsf{R}]\lambda^+,$$
$$\psi \ ::= \ \lambda_1 \land \dots \land \lambda_k \to \lambda^+ \ \mid \ \lambda_1 \land \dots \land \lambda_k \to \bot,$$
$$\varphi \ ::= \ p[m, n] \ \mid \ [G]\psi \ \mid \ \varphi_1 \land \varphi_2,$$

where p is a propositional variable, R any interval relation, $[\mathsf{R}]$ the dual of $\langle \mathsf{R} \rangle$, and $[G]$ the universal modality 'in all intervals'. Formulas of the form $p[m, n]$ are *initial clauses* (data) stating that p holds in the interval $[m, n]$; formulas of the form $[G]\psi$ are *universal clauses* describing general transformation rules and constraints; cf. (Fisher, Dixon, and Peim 2001). Our first result is that the satisfiability problem for $\mathcal{HS}_{horn}$ over the flow of time $\mathbb{Z}$ is P-complete (for both combined and data complexity) provided that the interpretation of the interval relations is *non-strict* (e.g., $[i, j]L[i', j']$ iff $j \leq i'$—rather than $j < i'$), which corresponds to the semantics of SQL:2011. Under the strict interpretation, $\mathcal{HS}_{horn}$ becomes PSPACE-hard. Note that the right-hand side of the implications ψ can only use 'boxes' $[\mathsf{R}]$. If 'diamonds' $\langle \mathsf{R} \rangle$ were also allowed, then the resulting fragment would be undecidable, as easily follows from the undecidability result of Bresolin, Muñoz-Velasco, and Sciavicco (2014).

Having identified a tractable fragment of $\mathcal{HS}$, we can use it as a template to define (hopefully tractable) temporal ontology languages. In this paper, we construct a temporalisation $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$ of the description logic *DL-Lite*$^{\mathcal{H}}_{horn}$ (Calvanese et al. 2007; Artale et al. 2009), which is a Horn extension of the ontology-based data access standard language OWL 2 QL[1]. In $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$, we represent temporal data by means of assertions such as

$$\textit{SummerSchool}(\textit{RW}, t_1, t_2), \quad \textit{teaches}(\textit{RK}, \textit{DL}, s_1, s_2),$$

[1] `www.w3.org/TR/owl2-profiles/#OWL_2_QL`

which say that RW is a summer school that takes place in the time interval $[t_1, t_2]$ and RK teaches DL in $[s_1, s_2]$. Note that temporal databases store data in a similar format (Kulkarni and Michels 2012). Temporal concept and role inclusions are used to impose various constraints on the data and introduce new concepts and roles. For example, according to

$$\langle \bar{D} \rangle \mathit{MorningSession} \sqcap \mathit{AdvancedCourse} \sqsubseteq \bot,$$

advanced courses cannot be given during the morning sessions; the axiom

$$\langle \bar{B} \rangle \mathit{LectureDay} \sqcap \langle A \rangle \mathit{Lunch} \sqsubseteq \mathit{MorningSession}$$

'defines' morning sessions (note that we are not allowed to replace $\sqsubseteq$ with $\equiv$ in this axiom). The inclusion

$$\mathit{teaches} \sqsubseteq [D]\mathit{teaches}$$

claims that the role *teaches* is downward hereditary (or stative) in the sense that if it holds in some interval, then it also holds in all of its sub-intervals. If, instead, we want to state that *teaches* is coalesced (or upward hereditary), in the sense that *teaches* holds in any interval covered by sub-intervals where it holds, then we can use

$$[D](\langle O \rangle \mathit{teaches} \sqcup \langle \bar{D} \rangle \mathit{teaches}) \sqcap \\ \langle B \rangle \mathit{teaches} \sqcap \langle E \rangle \mathit{teaches} \sqsubseteq \mathit{teaches}.$$

By removing the last two conjuncts on the left-hand side of this axiom, we make sure that *teaches* is both upward and downward hereditary. For a discussion of these notions in temporal databases, consult (Böhlen, Snodgrass, and Soo 1996; Terenziani and Snodgrass 2004).

Although the complexity of full $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{\mathit{horn}}$ remains unknown, in this paper we define two interesting fragments, for which instance checking is P-complete for both combined and data complexity. One fragment, $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}[G]}_{\mathit{horn}}$, restricts the use of temporal operators in role inclusion axioms, where only the 'universal' $[G]$ is allowed. The second one, $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$, allows only atomic concepts on the right-hand side of concept inclusions (but does not impose any restrictions on role inclusions).

The omitted proofs are available in (Artale et al. 2015).

Tractable $\mathcal{HS}_{\mathit{horn}}$

The syntax of the logic $\mathcal{HS}_{\mathit{horn}}$ was defied in the introduction. In this paper, we consider the interval relations A, $\bar{A}$, B, $\bar{B}$, E, $\bar{E}$, D, $\bar{D}$, L, $\bar{L}$, O, $\bar{O}$ and G over the set of closed intervals $[i, j] = \{n \in \mathbb{Z} \mid i \leq n \leq j\}$, for any integer numbers $i \leq j$, defined by taking:

– $[i, j]A[i', j']$ iff $j = i'$, (After)
– $[i, j]B[i', j']$ iff $i = i'$ and $j \geq j'$, (Begins)
– $[i, j]E[i', j']$ iff $i \leq i'$ and $j = j'$, (Ends)
– $[i, j]D[i', j']$ iff $i \leq i'$ and $j \geq j'$, (During)
– $[i, j]L[i', j']$ iff $j \leq i'$, (Later)
– $[i, j]O[i', j']$ iff $i \leq i' \leq j$ and $j \leq j'$, (Overlaps)

and $\bar{A}$, $\bar{B}$, $\bar{E}$, $\bar{D}$, $\bar{L}$, $\bar{O}$ to be the inverses of A, B, E, D, L, O, respectively. Note that we allow single-point intervals $[i, i]$ and use non-strict $\leq$ instead of the more common $<$.

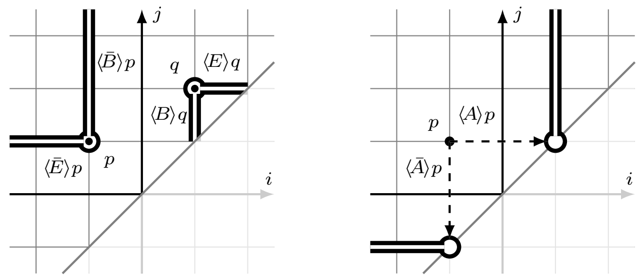

Figure 1: Semantics of the temporal operators: intervals $[i, j]$ are shown as points with the coordinates (i, j) and, e.g., if p is true in $[-1, 1]$ then $\langle \bar{E} \rangle p$ is true in all $[-k, 1]$, for $k \leq -1$.

Example 1. Consider the following $\mathcal{HS}_{\mathit{horn}}$-formula

$$\varphi = p[-1, 0] \wedge q[0, 0] \wedge q[0, 3] \wedge \\ [G](\langle E \rangle p \to q) \wedge [G]([\bar{A}]q \wedge q \to r).$$

The first three conjuncts—$p[-1, 0]$, $q[0, 0]$ and $q[0, 3]$—are called *initial clauses*: they state that p holds in $[-1, 0]$ and q in $[0, 0]$ and $[0, 3]$. The numbers occurring in the initial conditions are called *temporal constants* and given in binary.

An *interpretation*, $\mathfrak{M}$, for $\mathcal{HS}_{\mathit{horn}}$ assigns to every interval $[i, j]$ in $\mathbb{Z}$ a set of propositional variables, p, that are regarded to be *true* in $[i, j]$, in which case we write $\mathfrak{M}, [i, j] \models p$. This truth-relation is extended to $\mathcal{HS}_{\mathit{horn}}$-formulas by taking:

– $\mathfrak{M}, [i, j] \models p[m, n]$ iff $\mathfrak{M}, [m, n] \models p$,
– $\mathfrak{M}, [i, j] \models \langle \mathsf{R} \rangle \alpha$ iff $\mathfrak{M}, [i', j'] \models \alpha$ for some interval $[i', j']$ such that $[i, j]\mathsf{R}[i', j']$,
– $\mathfrak{M}, [i, j] \models [\mathsf{R}] \alpha$ iff $\mathfrak{M}, [i', j'] \models \alpha$ for all intervals $[i', j']$ such that $[i, j]\mathsf{R}[i', j']$,

and the usual clauses for the Booleans; see Fig. 1. An $\mathcal{HS}_{\mathit{horn}}$-formula φ is *satisfiable* if there is an interpretation $\mathfrak{M}$ such that $\mathfrak{M}, [0, 0] \models \varphi$; in this case we call $\mathfrak{M}$ a *model* of φ and write $\mathfrak{M} \models \varphi$. The length of φ is denoted by $|\varphi|$. Our main result in this section is a polynomial-time algorithm for checking satisfiability of $\mathcal{HS}_{\mathit{horn}}$-formulas (this problem is P-hard as the language contains propositional Horn clauses).

We represent any $\mathcal{HS}_{\mathit{horn}}$-formula φ as $\Xi \wedge \Psi^+ \wedge \Psi^-$, where Ξ is a conjunction of the initial clauses in φ and Ψ^+ (respectively, Ψ^-) is a conjunction of the universal clauses $[G]\psi$ in φ with λ^+ (respectively, $\bot$) on the right-hand side.

Lemma 2. *Any $\mathcal{HS}_{\mathit{horn}}$-formula can be transformed in polynomial time to an equisatisfiable formula $\Xi \wedge \Psi^+ \wedge \Psi^-$ such that it does not contain diamond operators, and its box operators only occur in contexts of the form $[G]\psi$ and $[\mathsf{R}]p$, where $\mathsf{R} \in \{A, \bar{A}, B, \bar{B}, E, \bar{E}, G\}$ and p a propositional variable.*

Proof. First, we express every $[\mathsf{R}]\lambda$ and $\langle \mathsf{R} \rangle \lambda$ in terms of the operators mentioned above: for instance, $[D]p$ is equivalent to $[B][E]p$ (see also Fig. 1). Then we replace every nested λ with a fresh variable p_λ and add $[G](\lambda \to p_\lambda)$ as a conjunct; we also replace every nested λ^+ with a fresh p_{λ^+} and add $[G](p_{\lambda^+} \to \lambda^+)$. Finally, we eliminate the diamonds by using the inverse relations: for instance, $[G](\langle E \rangle p \to q)$ from Example 1 is replaced with an equivalent $[G](p \to [\bar{E}]q)$. ❑

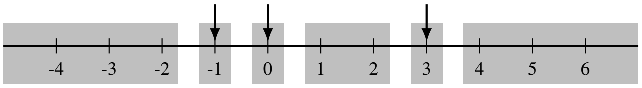

Figure 2: Partition of $\mathbb{Z}$ with respect to $\{-1, 0, 3\}$.

From now on we only consider $\mathcal{HS}_{horn}$-formulas of the form $\varphi = \Xi \wedge \Psi^+ \wedge \Psi^-$ given by Lemma 2. We now define a *canonical* (or *minimal*) *interpretation* $\mathfrak{K}_\varphi$ for φ by taking, for any variable p and any interval $[i, j]$,

$$\mathfrak{K}_\varphi, [i, j] \models p \quad \text{iff} \quad \mathfrak{M}, [i, j] \models p \text{ for all } \mathfrak{M} \models \Xi \wedge \Psi^+.$$

Clearly, $\Xi \wedge \Psi^+$ is always satisfiable.

Lemma 3. *For any formula $\varphi = \Xi \wedge \Psi^+ \wedge \Psi^-$, we have $\mathfrak{K}_\varphi \models \Xi \wedge \Psi^+$. Moreover, φ is satisfiable iff $\mathfrak{K}_\varphi \models \varphi$.*

Note that the Horn fragment of $\mathcal{HS}$ defined by (Bresolin, Muñoz-Velasco, and Sciavicco 2014) does not enjoy this minimal model property because $\langle \mathsf{R} \rangle$ on the right-hand side can represent disjunction; see (Artale et al. 2007, Theorem 11).

We now show how to construct efficiently the canonical interpretation for a given formula. We will require the following notation. Let $\mathbb{Z}^\omega = \mathbb{Z} \cup \{-\omega, \omega\}$ with $-\omega < i < \omega$, for any $i \in \mathbb{Z}$. Given $i, j \in \mathbb{Z}^\omega$, for $i \leq j$, we set $(i, j) = \{n \in \mathbb{Z} \mid i < n < j\}$. For a non-empty subset $M = \{m_0, m_1, \dots, m_n\}$ of $\mathbb{Z}$ with $m_0 < m_1 < \dots < m_n$, we define the *partition of $\mathbb{Z}$ with respect to M* to be the set $\mathbb{I}_M$ comprising the following intervals:

– $(-\omega, m_0)$, (m_n, ω) and $[m_k, m_k]$, for $0 \leq k \leq n$;

– (m_k, m_{k+1}), for $(m_k, m_{k+1}) \neq \emptyset$ and $0 \leq k < n$.

If $M = \emptyset$, we set $\mathbb{I}_M = \{(-\omega, \omega)\}$. Note that $I \cap I' = \emptyset$, for any distinct $I, I' \in \mathbb{I}_M$, and $\bigcup_{I \in \mathbb{I}_M} I = \mathbb{Z}$.

Given a formula $\varphi = \Xi \wedge \Psi^+ \wedge \Psi^-$, we denote by M_φ the set of integers occurring in Ξ. Note that $|\mathbb{I}_{M_\varphi}|$ is linear in $|\varphi|$: for instance, for φ from Example 1, $M_\varphi = \{-1, 0, 3\}$ and

$$\mathbb{I}_{M_\varphi} = \{(-\omega, -1), [-1, -1], [0, 0], (0, 3), [3, 3], (3, \omega)\},$$

see Fig. 2. The following lemma provides a key to the structure of canonical interpretations:

Lemma 4. *Let φ be an $\mathcal{HS}_{horn}$-formula, p a variable and $I, J \in \mathbb{I}_{M_\varphi}$. If there exist $i \in I$ and $j \in J$ with $i \leq j$ and $\mathfrak{K}_\varphi, [i, j] \models p$, then $\mathfrak{K}_\varphi, [i', j'] \models p$ for all $i' \in I$ and $j' \in J$ with $i' \leq j'$.*

This lemma shows that the canonical interpretation for φ can be constructed by applying the rules in Ψ^+ to the closed intervals in the *finite* linear order $(\mathbb{I}_{M_\varphi}, \preceq)$, where $\preceq$ is defined by taking $I \preceq J$, for $I, J \in \mathbb{I}_M$, iff $i \leq j$, for some $i \in I$ and $j \in J$. The interval relations $[I, J]\mathsf{R}[I', J']$ in $(\mathbb{I}_{M_\varphi}, \preceq)$, for $\mathsf{R} \in \{A, \bar{A}, B, \bar{B}, E, \bar{E}, G\}$, are defined as usual. Now, the canonical interpretation for $\varphi = \Xi \wedge \Psi^+ \wedge \Psi^-$ can be constructed using the following *chase procedure*. We first set $\mathfrak{C}_\varphi = \{(p, [m, m], [n, n]) \mid p[m, n] \in \Xi\}$ and then apply to $\mathfrak{C}_\varphi$ the following rules: Suppose $(p_k, I, J) \in \mathfrak{C}_\varphi$, for $1 \leq k \leq h$, and $(q_k, I', J') \in \mathfrak{C}_\varphi$, for all $I', J' \in \mathbb{I}_{M_\varphi}$ with $[I, J]\mathsf{R}_k[I', J']$ and $1 \leq k \leq \ell$, then

– If $p_1 \wedge \dots \wedge p_h \wedge [\mathsf{R}_1]q_1 \wedge \dots \wedge [\mathsf{R}_\ell]q_\ell \to p$ is in Ψ^+ then $\mathfrak{C}_\varphi := \mathfrak{C}_\varphi \cup \{(p, I, J)\}$.

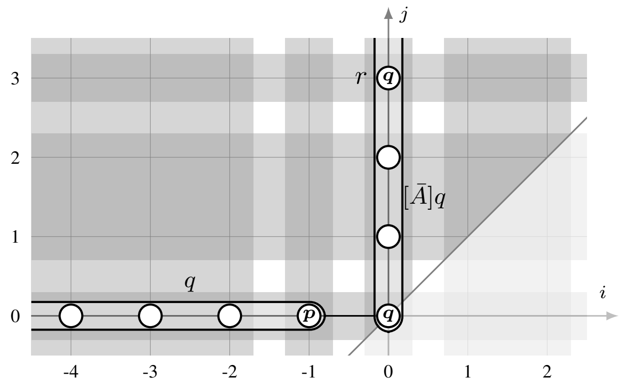

Figure 3: Canonical interpretation from Example 5.

– If $p_1 \wedge \dots \wedge p_h \wedge [\mathsf{R}_1]q_1 \wedge \dots \wedge [\mathsf{R}_\ell]q_\ell \to [\mathsf{R}]p$ is in Ψ^+ then $\mathfrak{C}_\varphi := \mathfrak{C}_\varphi \cup \{(p, I', J') \mid [I, J]\mathsf{R}[I', J']\}$.

As $\mathfrak{C}_\varphi \subseteq \mathsf{V}_\varphi \times \mathbb{I}_{M_\varphi} \times \mathbb{I}_{M_\varphi}$, where V_φ is the set of variables in φ, and $|\mathbb{I}_{M_\varphi}| = O(|\varphi|)$, the chase procedure reaches a fixed point $\mathfrak{C}^*_\varphi$ after *polynomially many* steps. The canonical interpretation $\mathfrak{K}_\varphi$ is now defined by taking $\mathfrak{K}_\varphi, [i, j] \models p$ iff $(p, I, J) \in \mathfrak{C}^*_\varphi$, $i \in I$ and $j \in J$.

Example 5. The canonical interpretation $\mathfrak{K}_\varphi$ for φ from Example 1 can be defined by taking (see Fig. 3):

$$\begin{aligned}
&\mathfrak{K}_\varphi, [i, j] \models p \quad &&\text{iff} \quad i = -1,\ j = 0,\\
&\mathfrak{K}_\varphi, [i, j] \models q \quad &&\text{iff} \quad i \leq 0,\ j = 0 \ \text{ or } \ i = 0,\ j = 3,\\
&\mathfrak{K}_\varphi, [i, j] \models r \quad &&\text{iff} \quad i = 0,\ j = 0 \ \text{ or } \ i = 0,\ j = 3.
\end{aligned}$$

As a consequence of the (polynomial) construction of the canonical interpretations and Lemma 3, we obtain:

Theorem 6. *The satisfiability problem for $\mathcal{HS}_{horn}$-formulas is* P*-complete.*

Another consequence of the construction (which will be used in the proof of Lemma 13) is that satisfiability of $\mathcal{HS}_{horn}$-formulas is preserved under the following scaling of initial clauses. Let $\varphi_i = \Xi_i \wedge \Psi^+ \wedge \Psi^-$ and $M_{\varphi_i} = \{m^i_0, \dots, m^i_n\}$ with $m^i_0 < \dots < m^i_n$, for $i = 1, 2$. We say that φ_1 and φ_2 are *variants* if

– $\min\{m^1_{k+1} - m^1_k, 2\} = \min\{m^2_{k+1} - m^2_k, 2\}$, for any $0 \leq k < n$,

– $p[m^1_i, m^1_j] \in \Xi_1$ iff $p[m^2_i, m^2_j] \in \Xi_2$, for any variable p.

The former condition means that the difference between the adjacent m^i_k is the same for both i modulo counting in terms of 0, 1 and *many*. It follows, in particular, that $(\mathbb{I}_{M_{\varphi_1}}, \preceq)$ and $(\mathbb{I}_{M_{\varphi_2}}, \preceq)$ are isomorphic as linear orders (see also Example 8). The two conditions together imply the following:

Lemma 7. *Any two variants are equisatisfiable.*

Example 8. To see that we need all of 0, 1 and 2 in the first condition above, take $\varphi = [G](p \wedge [E]p \to \bot)$ and $\psi = [G](p \to q \wedge [A]q) \wedge [G](p \wedge [E]q \to \bot)$. Then the formulas $p[0, 1] \wedge \varphi$ and $p[0, 2] \wedge \psi$ are satisfiable, while $p[0, 0] \wedge \varphi$ and $p[0, 1] \wedge \psi$ are not.

Remark 9. If in place of $\leq$ in the definition of the interval relations we take $<$, then reasoning with $\mathcal{HS}_{horn}$ becomes non-tractable (unless P = PSPACE). Indeed, given a Turing machine $\mathfrak{A}$ with polynomial tape, we take the following initial clauses: $a_i[0,0]$ to say that input cell i contains a, $h_0[0,0]$ to indicate that the head scans the left-most cell, and $q_0[0,0]$ to fix the initial state q_0. Instructions such as $(q,a) \to (q', a', R)$ are encoded by formulas of the form

$$[G]\big(\langle\bar{E}\rangle[B](q \wedge a_i \wedge h_i) \to q' \wedge a'_i \wedge h_{i+1}\big).$$

(Thus, we represent the consecutive configurations of $\mathfrak{A}$ on the 'diagonal intervals' $[n,n]$, $n \geq 0$, using the 'previous-time' operator $\langle\bar{E}\rangle[B]$.) Then $\mathfrak{A}$ accepts the input iff the conjunction of the above formulas and $[G](q_1 \to \bot)$, for the accepting state q_1, is unsatisfiable. At the moment, the exact complexity of $\mathcal{HS}_{horn}$ under the strict semantics is not known. Note that the full $\bar{E}B$-fragment of $\mathcal{HS}$ is undecidable (Bresolin et al. 2014a).

Data Complexity

As $\mathcal{HS}_{horn}$-formulas consist of initial clauses (that is, *data*) and universal clauses, we can also measure the complexity of the satisfiability problem in terms of the size of the data regarding the universal clauses fixed.

Theorem 10. *$\mathcal{HS}_{horn}$ is* P*-complete for data complexity.*

Proof. Theorem 6 gives the upper bound. The proof of hardness is by a LOGSPACE-reduction of the monotone circuit value problem, which is known to be P-complete; see e.g., (Greenlaw, Hoover, and Ruzzo 1995; Miyano, Shiraishi, and Shoudai 1990). Suppose $\boldsymbol{C}$ is a monotone circuit whose vertices (sources, gates and sink) are enumerated by consecutive positive integers in such a way that if there is an edge from a vertex n to a vertex m—in which case we write $n \rightsquigarrow m$—then $n < m$. Denote by $\max \boldsymbol{C}$ the maximum of the vertex numbers. We can assume that $\max \boldsymbol{C}$ is the sink of $\boldsymbol{C}$ (so $\max \boldsymbol{C} - 1 \rightsquigarrow \max \boldsymbol{C}$). We represent $\boldsymbol{C}(\vec{x})$, for an input $\vec{x}$, by the conjunction $\Xi_{\boldsymbol{C}(\vec{x})}$ of the following initial clauses:

- $t[\max \boldsymbol{C} - 1, \max \boldsymbol{C}]$,
- $t[n,m]$ (or $f[n,m]$) if n is a source with input value 1 (respectively, 0) and $n \rightsquigarrow m$;
- AND$[n,m]$ (or OR$[n,m]$) if n is an AND gate (respectively, OR gate) and $n \rightsquigarrow m$;
- *and*$[0,m]$ and $t[n,m]$, for each n such that $0 < n \leq m$ and $n \not\rightsquigarrow m$, if m is an AND gate;
- *or*$[0,m]$ and $f[n,m]$, for each n such that $0 < n \leq m$ and $n \not\rightsquigarrow m$, if m is an OR gate.

Let Ψ^+ be a conjunction of the following universal clauses:

$$[G](\langle\bar{A}\rangle f \wedge \text{AND} \to f), \qquad [G]([\bar{A}] f \wedge \text{OR} \to f),$$
$$[G]([\bar{A}] t \wedge \text{AND} \to t), \qquad [G](\langle\bar{A}\rangle t \wedge \text{OR} \to t),$$
$$[G](\mathit{and} \to [\bar{E}]t), \qquad [G](\mathit{or} \to [\bar{E}]f),$$

and let $\Psi^- = [G](t \wedge f \to \bot)$. It is not hard to check that $\Xi_{\boldsymbol{C}(\vec{x})} \wedge \Psi^+ \wedge \Psi^-$ is satisfiable iff $\boldsymbol{C}(\vec{x}) = 1$. (Intuitively, the last items in the definitions of the initial and universal clauses ensure that, for any OR gate m, all intervals of the form $[n,m]$ with $n \not\rightsquigarrow m$ are labelled with f; and dually the AND gates with t. Thus, the output of any gate only depends on its inputs.) ❑

It is of interest to note that a similar Horn fragment of the point-based *LTL* is in AC^0 for data complexity, while the whole *LTL* is NC^1-complete (Artale et al. 2014a); we remind the reader that $\mathrm{AC}^0 \subsetneq \mathrm{NC}^1 \subseteq \mathrm{P}$.

We now use $\mathcal{HS}_{horn}$ as a template for defining a temporal extension of the description logic *DL-Lite*$^{\mathcal{H}}_{horn}$ with the ultimate aim of employing it or its suitable fragments for ontology-based data access over temporal databases.

Description Logic $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$

The language of $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$ contains *individual names* $a_0, a_1, \dots$, *concept names* $A_0, A_1, \dots$, and *role names* $P_0, P_1, \dots$. *Basic roles* R, *basic concepts* B, *temporal roles* S and *temporal concepts* C are given by the following grammar:

$$R \ ::= \ P_k \ | \ P_k^-, \qquad B \ ::= \ A_k \ | \ \exists R,$$
$$S \ ::= \ R \ | \ [\mathsf{R}]S, \qquad C \ ::= \ B \ | \ [\mathsf{R}]C,$$

where R is one of the interval relations. An $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$ TBox is a finite set of *concept* and *role inclusions*

$$C_1 \sqcap \dots \sqcap C_k \sqsubseteq C, \qquad S_1 \sqcap \dots \sqcap S_k \sqsubseteq S,$$

and *disjointness constraints*

$$C_1 \sqcap \dots \sqcap C_k \sqsubseteq \bot, \qquad S_1 \sqcap \dots \sqcap S_k \sqsubseteq \bot.$$

Note that, similarly to Lemma 2, we could also allow the diamond operators $\langle\mathsf{R}\rangle C$ and $\langle\mathsf{R}\rangle S$ on the *left-hand side* of concept and role inclusions and disjointness constraints. They are omitted to simplify presentation.

An $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$ ABox is a finite set of atoms of the form $A_k(a,i,j)$ and $P_k(a,b,i,j)$ in which *temporal constants* $i \leq j$ are given in binary. The set of individual names in $\mathcal{A}$ is denoted by *ind*$(\mathcal{A})$. An $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$ *knowledge base* (KB) is a pair $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is a TBox and $\mathcal{A}$ an ABox.

An $\mathcal{HS}$-*Lite*$^{\mathcal{H}}_{horn}$ *interpretation*, $\mathcal{I}$, consists of a family of standard (atemporal) description logic interpretations $\mathcal{I}[i,j] = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}[i,j]})$, for all $i,j \in \mathbb{Z}$ with $i \leq j$, in which $\Delta^{\mathcal{I}} \neq \emptyset$, $a_k^{\mathcal{I}[i,j]} = a_k^{\mathcal{I}}$ for some (fixed) $a_k^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, $\bot^{\mathcal{I}[i,j]} = \emptyset$, $A_k^{\mathcal{I}[i,j]} \subseteq \Delta^{\mathcal{I}}$ and $P_k^{\mathcal{I}[i,j]} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The role and concept constructs are interpreted in $\mathcal{I}$ as follows:

$$(P_k^-)^{\mathcal{I}[i,j]} \ = \ \{(x,y) \mid (y,x) \in P_k^{\mathcal{I}[i,j]}\},$$
$$(\exists R)^{\mathcal{I}[i,j]} \ = \ \{x \mid (x,y) \in R^{\mathcal{I}[i,j]}, \text{ for some } y \in \Delta^{\mathcal{I}}\},$$
$$([\mathsf{R}]C)^{\mathcal{I}[i,j]} \ = \ \textstyle\bigcap_{[i,j]\mathsf{R}[i',j']} C^{\mathcal{I}[i',j']},$$
$$([\mathsf{R}]S)^{\mathcal{I}[i,j]} \ = \ \textstyle\bigcap_{[i,j]\mathsf{R}[i',j']} S^{\mathcal{I}[i',j']}.$$

The *satisfaction relation* $\models$ is defined by taking:

$$\mathcal{I} \models A(a,i,j) \quad \text{iff} \quad a^{\mathcal{I}} \in A^{\mathcal{I}[i,j]},$$
$$\mathcal{I} \models P(a,b,i,j) \quad \text{iff} \quad (a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}[i,j]},$$
$$\mathcal{I} \models \textstyle\bigsqcap_k C_k \sqsubseteq C \quad \text{iff} \quad \textstyle\bigcap_k C_k^{\mathcal{I}[i,j]} \subseteq C^{\mathcal{I}[i,j]}, \ \textit{for all } [i,j],$$
$$\mathcal{I} \models \textstyle\bigsqcap_k S_k \sqsubseteq S \quad \text{iff} \quad \textstyle\bigcap_k S_k^{\mathcal{I}[i,j]} \subseteq S^{\mathcal{I}[i,j]}, \ \textit{for all } [i,j],$$

and similarly for disjointness constraints. Note that concept and role inclusions as well as disjointness constraints are interpreted *globally*. For a TBox inclusion or an ABox assertion α, we write $\mathcal{K} \models \alpha$ if $\mathcal{I} \models \alpha$, for all models $\mathcal{I}$ of $\mathcal{K}$ (that is, for all $\mathcal{I}$ with $\mathcal{I} \models \mathcal{K}$). Similarly, we write $\mathcal{T} \models \alpha$ in case $(\mathcal{T}, \emptyset) \models \alpha$.

The complexity of reasoning with $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}}_{\mathit{horn}}$ is still unknown. Our aim in the remainder of this paper is to show that two of its fragments are tractable. The first fragment only allows those $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}}_{\mathit{horn}}$ TBoxes that are *flat* in the sense that their concept inclusions do not contain $\exists R$ on the right-hand side. We denote this fragment by $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$.

Tractability of $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$

We show that, for any $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$ KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, one can construct in polynomial time an equisatisfiable $\mathcal{HS}_{\mathit{horn}}$-formula $\varphi_{\mathcal{K}}$.

We require the following notation. For a basic role R, we set $R^- = P_k^-$ if $R = P_k$, and $R^- = P_k$ if $R = P_k^-$. Given an $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}}_{\mathit{horn}}$ TBox $\mathcal{T}$, we denote by $\mathit{rol}(\mathcal{T})$ the set of basic roles R such that R or R^- occurs in $\mathcal{T}$, and by $\mathit{con}(\mathcal{T})$ the set of basic concepts B occurring in $\mathcal{T}$ as well as all basic concepts $\exists R$, for $R \in \mathit{rol}(\mathcal{T})$.

Theorem 11. *The satisfiability problem for $\mathcal{HS}$-Lite$^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$ KBs is* P*-complete.*

Proof. P-hardness is from the propositional Horn logic. The matching upper bound proof is by a polynomial-time reduction to $\mathcal{HS}_{\mathit{horn}}$. Given a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, take propositional variables $p^{B,a}$ and $p^{R,a,b}$, for any $B \in \mathit{con}(\mathcal{T})$, $R \in \mathit{rol}(\mathcal{T})$ and $a, b \in \mathit{ind}(\mathcal{A})$. For any concept $C = [\mathsf{R}_1] \dots [\mathsf{R}_n]B$ in $\mathcal{T}$ and $a \in \mathit{ind}(\mathcal{A})$, let $C^a = [\mathsf{R}_1] \dots [\mathsf{R}_n]p^{B,a}$; similarly, for any role S in $\mathcal{T}$ and $a, b \in \mathit{ind}(\mathcal{A})$, define $S^{a,b}$ using $p^{S,a,b}$. Let $\varphi_{\mathcal{K}}$ be a conjunction of the following $\mathcal{HS}_{\mathit{horn}}$-formulas:

- $p^{A,a}[i,j]$, for $A(a,i,j) \in \mathcal{A}$,
- $p^{P,a,b}[i,j]$, for $P(a,b,i,j) \in \mathcal{A}$,
- $[G](p^{R,a,b} \to p^{R^-,b,a})$ and $[G](p^{R,a,b} \to p^{\exists R,a})$, for any $R \in \mathit{rol}(\mathcal{T})$ and $a, b \in \mathit{ind}(\mathcal{A})$,
- $[G](\bigwedge_k C_k^a \to C^a)$, for $\bigsqcap_k C_k \sqsubseteq C$ in $\mathcal{T}$ and $a \in \mathit{ind}(\mathcal{A})$,
- $[G](\bigwedge_k S_k^{a,b} \to S^{a,b})$, for $\bigsqcap_k S_k \sqsubseteq S$ in $\mathcal{T}$, $a,b \in \mathit{ind}(\mathcal{A})$,

and similar formulas for the disjointness constraints in $\mathcal{T}$. One can now show that $\varphi_{\mathcal{K}}$ is equisatisfiable with $\mathcal{K}$. ❑

Similarly to the canonical interpretations for $\mathcal{HS}_{\mathit{horn}}$-formulas, we now define canonical interpretations for $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$ KBs, which will be used in the next section. Given a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, we denote by $\mathcal{T}^+$ the set of concept and role inclusions in $\mathcal{T}$ and by $\mathcal{T}^-$ the set of disjointness constraints in $\mathcal{T}$. A *canonical interpretation* $\mathfrak{K}_{\mathcal{K}} = (\Delta^{\mathfrak{K}_{\mathcal{K}}}, \cdot^{\mathfrak{K}_{\mathcal{K}}})$ for $\mathcal{K}$ is defined by taking, for $a, b \in \mathit{ind}(\mathcal{A})$ and $i \le j$,

- $\Delta^{\mathfrak{K}_{\mathcal{K}}} = \mathit{ind}(\mathcal{A})$ and $a^{\mathfrak{K}_{\mathcal{K}}} = a$,
- $a \in A^{\mathfrak{K}_{\mathcal{K}}[i,j]}$ iff $(\mathcal{T}^+, \mathcal{A}) \models A(a,i,j)$,
- $(a,b) \in P^{\mathfrak{K}_{\mathcal{K}}[i,j]}$ iff $(\mathcal{T}^+, \mathcal{A}) \models P(a,b,i,j)$

(see Example 12 below). Similarly to Lemma 3, one can show that $\mathfrak{K}_{\mathcal{K}} \models (\mathcal{T}^+, \mathcal{A})$ and $\mathcal{K}$ is satisfiable iff $\mathfrak{K}_{\mathcal{K}} \models \mathcal{K}$.

Tractability of $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}[G]}_{\mathit{horn}}$

Our second fragment, denoted $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}[G]}_{\mathit{horn}}$, allows only the operator $[G]$ in the definition of temporal roles S (with no restrictions imposed on temporal concepts). Thus, unlike $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{\mathit{horn}}$, the fragment $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}[G]}_{\mathit{horn}}$ contains full DL-$\mathit{Lite}^{\mathcal{H}}_{\mathit{horn}}$. We now show that reasoning with this fragment is also tractable. For any role name P, we reserve two special concept names, EP and EP^-.

Given an $\mathcal{HS}$-$\mathit{Lite}^{\mathcal{H}[G]}_{\mathit{horn}}$ TBox $\mathcal{T}$, we define the *flattening* of $\mathcal{T}$ to be the TBox $\mathcal{T}' = \mathcal{T}_1 \cup \mathcal{T}_2$, where $\mathcal{T}_1$ results from $\mathcal{T}$ by replacing every $\exists R$ with ER, and $\mathcal{T}_2$ comprises

$$\begin{aligned} \exists R &\sqsubseteq ER, \\ ER &\sqsubseteq EQ, \quad \text{if } \mathcal{T} \models R \sqsubseteq Q, \\ ER &\sqsubseteq [G]EQ, \quad \text{if } \mathcal{T} \models R \sqsubseteq [G]Q, \end{aligned}$$

for all $R, Q \in \mathit{rol}(\mathcal{T})$. Clearly, $\mathcal{T}'$ is flat and, by Theorem 11, can be computed in polynomial time.

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a KB. For any $\delta \in \{0, 1, 2\}$ (where 2 stands for 'many'; see Lemma 7) and $R \in \mathit{rol}(\mathcal{T})$, let d^R_δ be a fresh individual name. Let $\mathcal{T}'$ be the flattening of $\mathcal{T}$. Given an extension $\mathcal{A}'$ of $\mathcal{A}$ with some atoms of the form $P(d^P_\delta, d^{P^-}_\delta, 0, \delta)$, for $\delta \in \{0, 1, 2\}$, let $\mathfrak{K}' = (\Delta^{\mathfrak{K}'}, \cdot^{\mathfrak{K}'})$ be the canonical interpretation for $\mathcal{K}' = (\mathcal{T}', \mathcal{A}')$. We call $\mathcal{A}'$ a *witness ABox* for $\mathcal{K}$ in case the following is satisfied:

(witn) if $EP^{\mathfrak{K}'[i,j]} \neq \emptyset$ or $(EP^-)^{\mathfrak{K}'[i,j]} \neq \emptyset$, for some $i \le j$, then $P(d^P_\delta, d^{P^-}_\delta, 0, \delta) \in \mathcal{A}'$, where $\delta = \min\{j - i, 2\}$.

This condition ensures that, for each role name P with nonempty EP or EP^-, we have *witnesses* d^P_δ and $d^{P^-}_\delta$ in the intervals of length 0, 1 or 2. By Lemma 7, witnesses for P in intervals of length greater than 2 can be obtained from the witnesses for length 2.

Example 12. Suppose $\mathcal{T} = \{A \sqsubseteq \exists P,\ \exists P^- \sqsubseteq [\bar{B}]\exists P\}$ and $\mathcal{A} = \{A(a, -1, 3)\}$. Then $\mathcal{T}'$ consists of

$$A \sqsubseteq EP,\ \ EP^- \sqsubseteq [\bar{B}]EP,\ \ \exists P \sqsubseteq EP,\ \ \exists P^- \sqsubseteq EP^-.$$

Let $\mathcal{A}' = \{A(a, -1, 3), P(d^P_2, d^{P^-}_2, 0, 2)\}$. Then the canonical interpretation $\mathfrak{K}' = (\Delta^{\mathfrak{K}'}, \cdot^{\mathfrak{K}'})$ of $(\mathcal{T}', \mathcal{A}')$ is as follows:

- $\Delta^{\mathfrak{K}'} = \{a, d^P_2, d^{P^-}_2\}$;
- $A^{\mathfrak{K}'[-1,3]} = \{a\}$; otherwise, $A^{\mathfrak{K}'[i,j]} = \emptyset$;
- $P^{\mathfrak{K}'[0,2]} = \{(d^P_2, d^{P^-}_2)\}$; otherwise, $P^{\mathfrak{K}'[i,j]} = \emptyset$;
- $EP^{\mathfrak{K}'[-1,3]} = \{a\}$, $EP^{\mathfrak{K}'[0,2]} = \{d^P_2, d^{P^-}_2\}$ and, for any $k \ge 2$, $EP^{\mathfrak{K}'[0,k]} = \{d^{P^-}_2\}$; otherwise, $EP^{\mathfrak{K}'[i,j]} = \emptyset$;
- $(EP^-)^{\mathfrak{K}'[0,2]} = \{d^{P^-}_2\}$; otherwise, $(EP^-)^{\mathfrak{K}'[i,j]} = \emptyset$.

Thus, $\mathcal{A}'$ is a witness ABox for $\mathcal{K} = (\mathcal{T}, \mathcal{A})$.

We can now unravel $\mathfrak{K}'$ into a model $\mathcal{I}$ of $\mathcal{K}$ by constructing a sequence of interpretations $\mathcal{I}_k = (\Delta^{\mathcal{I}_k}, \cdot^{\mathcal{I}_k})$, where $\mathcal{I}_{k+1}$ extends $\mathcal{I}_k$, and setting $\mathcal{I} = \bigcup_{k \ge 0} \mathcal{I}_k$. First we use $\mathfrak{K}'$ to define $\mathcal{I}_0$ by taking $\Delta^{\mathcal{I}_0} = \{a\}$, $A^{\mathcal{I}_0[-1,3]} = \{a\}$, $A^{\mathcal{I}_0[i,j]} = \emptyset$ for all other intervals, and $P^{\mathcal{I}_0[i,j]} = \emptyset$ for all $[i,j]$. We then observe that, in $\mathcal{I}_0$, a has a 'defect' in the interval $[-1,3]$ because it does not have a P-successor required by $a \in (EP)^{\mathfrak{K}'[-1,3]}$. We 'cure' this defect in the

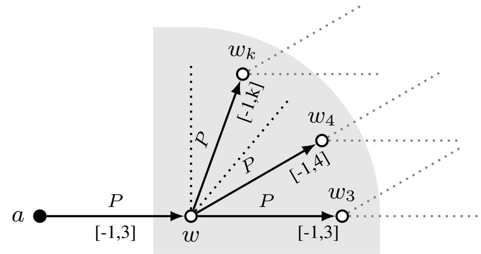

Figure 4: The unravelling construction.

interpretation $\mathcal{I}_1$ by adding to $\Delta^{\mathcal{I}_0}$ a (fresh) copy w of $d_2^{P^-}$ and setting $P^{\mathcal{I}_1[-1,3]} = \{(a, w)\}$ and $P^{\mathcal{I}_1[i,j]} = \emptyset$ for all other intervals. But now the newly introduced element w in $\mathcal{I}_1$ has a defect in every interval $[-1, k]$, $k \geq 3$, because it does not have a required P-successor: indeed, $d_2^{P^-}$ belongs to EP in every such interval in the canonical interpretation of $(\mathcal{T}', P(d_2^P, d_2^{P^-}, -1, 3))$. To cure those defects, we add to $\mathcal{I}_1$ fresh copies w_k of $d_2^{P^-}$, for $k \geq 3$, and then set $P^{\mathcal{I}_2[-1,k]} = \{(w, w_k)\}$ and $P^{\mathcal{I}_2[i,j]} = \emptyset$ for all other intervals, and so forth (see Fig. 4). The proof of Theorem 13 below shows that $\mathcal{I} \models \mathcal{K}$ iff $\mathfrak{K}' \models \mathcal{T}'$.

Clearly, any KB $\mathcal{K}$ has at least one witness ABox.

Theorem 13. *$\mathcal{K}$ is satisfiable iff there exists a witness ABox $\mathcal{A}'$ for $\mathcal{K}$ such that $\mathfrak{K}' \models \mathcal{T}'$.*

The proof of ($\Leftarrow$), illustrated by Example 12, uses an unravelling technique similar to that of (Artale et al. 2014b, Theorem 4.1 and Lemma 6.5) developed for point-based temporal *DL-Lite*. An essential difference from the earlier construction is that now we not only *shift* the interpretation underlying the timeline of witnesses d_δ^R in order to cure a defect, but also *stretch* (using Lemma 7) some intervals in these interpretations.

To show ($\Rightarrow$), we first construct a *minimal* witness ABox $\mathcal{A}'$ for $\mathcal{K}$ by taking $(\mathcal{T}', \mathcal{A})$ and recursively adding to $\mathcal{A}$ the missing witnesses $P(d_\delta^P, d_\delta^{P^-}, 0, \delta)$. In fact, we prove that a fixed point in this construction will be reached in polynomially many steps. Then we consider the unravelling of the canonical interpretation $\mathfrak{K}'$ for $\mathcal{K}' = (\mathcal{T}', \mathcal{A}')$ and show that it is homomorphically embeddable into any model of $\mathcal{K}$. That $\mathfrak{K}' \models \mathcal{T}'$ follows now by the construction of the unravelling.

As a consequence of this proof we finally obtain:

Theorem 14. *The satisfiability problem for $\mathcal{HS}$-Lite$_{horn}^{\mathcal{H}[G]}$ KBs is* P*-complete.*

Remark 15. Unfortunately, the construction above does not work for the whole $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}}$, where arbitrary operators [R] can be used in the definition of temporal roles S. To see why, consider first the $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}[G]}$ KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ with $\mathcal{T} = \{\ A \sqsubseteq \exists P,\ \ P \sqsubseteq [G]S\ \}$ and $\mathcal{A} = \{A(a, 0, 0)\}$. Then $a^{\mathcal{I}} \in (\exists S)^{\mathcal{I}[i,j]}$ for any $\mathcal{I} \models \mathcal{K}$ and $i \leq j$. The axioms of $\mathcal{T}_2$ make sure that $a^{\mathcal{I}} \in ES^{\mathcal{I}[i,j]}$.

Consider now the $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}}$ KB $\mathcal{K}' = (\mathcal{T}', \mathcal{A})$ with $\mathcal{T} = \{A \sqsubseteq [G]\exists P,\ P \sqsubseteq [A]P_1,\ P \sqsubseteq [\bar{A}]P_2,\ P_1 \sqcap P_2 \sqsubseteq S\}$. We then have $a^{\mathcal{I}} \in (\exists S)^{\mathcal{I}[i,i]}$, for any $\mathcal{I} \models \mathcal{K}'$ and $i \in \mathbb{Z}$. However, it is not clear what axioms of $\mathcal{T}_2$ could make sure that $a^{\mathcal{I}} \in ES^{\mathcal{I}[i,i]}$.

Data Complexity of Instance Checking

One of the main reasoning problems in description logic is *instance checking*. In our context it can be formulated as follows: given a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and an atom $C(a, i, j)$, where C is a concept, a an individual name and $i \leq j$, decide whether $\mathcal{K} \models C(a, i, j)$. As instance checking is reducible to satisfiability, it is P-complete for both $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}/flat}$ and $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}[G]}$ for combined complexity. Moreover, as a consequence of Theorems 10, 11 and 14, we also obtain:

Theorem 16. *Instance checking for both $\mathcal{HS}$-Lite$_{horn}^{\mathcal{H}/flat}$ and $\mathcal{HS}$-Lite$_{horn}^{\mathcal{H}[G]}$ is* P*-complete for data complexity* (*when only the ABox is regarded to be the input*).

This result contrasts with the lower data complexity (AC^0 and NC^1) of instance checking with point-based temporal *DL-Lite* (Artale et al. 2013; 2014a).

Outlook

Our interest in tractable description logics with interval-based temporal operators is motivated by possible applications in ontology-based data access (OBDA) over temporal databases. In the OBDA paradigm, one can query data sources, D, using the vocabulary of an ontology, $\mathcal{T}$, that provides a unifying conceptual view of the data and enriches it with background knowledge (Calvanese et al. 2007). Given a query, $\boldsymbol{q}$, an OBDA system rewrites $\boldsymbol{q}$ and $\mathcal{T}$ into another query, $\boldsymbol{q}'$, such that $\mathcal{T}, D \models \boldsymbol{q}$ iff $D \models \boldsymbol{q}'$, for any data D. A standard ontology language that guarantees the existence of a first-order rewriting $\boldsymbol{q}'$ is the OWL 2 QL profile of the Web Ontology Language OWL 2. (In a nutshell, OWL 2 QL is *DL-Lite*$_{horn}^{\mathcal{H}}$ in which concept and role inclusions cannot have $\sqcap$ on the left-hand side.) In the context of *temporal databases*, we are interested in suitable ontology and query languages with temporal constructs (although some authors advocate the use of standard OWL 2 QL with temporal queries (Klarman 2014; Borgwardt, Lippmann, and Thost 2013)).

As modern temporal databases adopt the (downward hereditary) interval-based model of time (Kulkarni and Michels 2012) and use coalescing to group time points into intervals (Böhlen, Snodgrass, and Soo 1996), in this paper we have launched an investigation of ontology languages that can be suitable for OBDA over such databases by designing the language $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}}$ and its tractable fragments $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}/flat}$ and $\mathcal{HS}$-*Lite*$_{horn}^{\mathcal{H}[G]}$. In view of Theorem 16, these languages cannot guarantee first-order rewritability of even atomic queries, though we believe datalog rewritings are possible. We leave the query rewritability issues, in particular, the design of *DL-Lite*$_{core}^{\mathcal{H}}$-based fragments supporting first-order rewritability as well as temporal extensions of the OWL 2 EL and OWL 2 RL profiles of OWL 2 for future research.

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