# The *DL-Lite* Family and Relations

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joint work with

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# Motivating example: DL for Conceptual Modelling

## Translating into DL:

$\mathsf{TopManager} \sqsubseteq \mathsf{Manager}$

$\mathsf{AreaManager} \sqsubseteq \neg\mathsf{TopManager}$

$\mathsf{Manager} \sqsubseteq \mathsf{AreaManager} \sqcup \mathsf{TopManager}$

$\mathsf{Employee} \sqsubseteq \exists \mathit{salary}.\top$

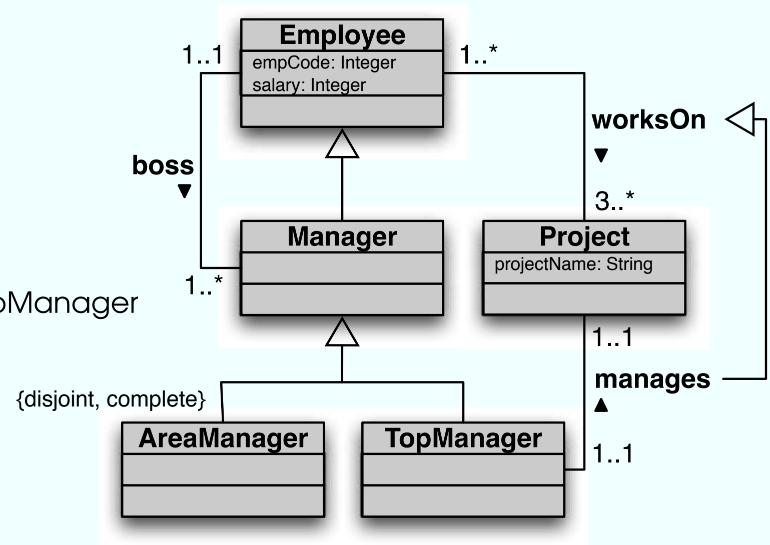
$\exists \mathit{salary}^{-}.\top \sqsubseteq \mathsf{Integer}$

$\geq 2\ \mathit{salary}.\top \sqsubseteq \bot$

$\mathsf{Project} \sqsubseteq \ \geq 3\ \mathit{worksOn}^{-}.\top$

$\mathsf{CEO} \sqcap (\geq 5\ \mathit{worksOn}.\top) \sqcap \exists \mathit{manages}.\top \sqsubseteq \bot$

$\mathit{manages} \sqsubseteq \mathit{worksOn}$




**DL-Lite!**

# The *DL-Lite* family

1. $\mathit{DL\text{-}Lite}^{\mathcal{N}}_{bool}$

combined complexity sat.: **NP**
data comp. instance: in **$AC^0$**
data comp. query: **coNP**

$$\begin{array}{lllllll} R & ::= & P & \mid & P^- & & \\ B & ::= & \bot & \mid & A & \mid & \geq qR \\ C & ::= & B & \mid & \neg C & \mid & C_1 \sqcap C_2 \end{array}$$

TBox axioms $C_1 \sqsubseteq C_2$

2. $\mathit{DL\text{-}Lite}^{\mathcal{N}}_{horn}$

combined complexity: **P**
data comp. instance: in **$AC^0$**
data comp. query: in **$AC^0$**

TBox axioms $B_1 \sqcap \cdots \sqcap B_n \sqsubseteq B$

3. $\mathit{DL\text{-}Lite}^{\mathcal{N}}_{krom}$

comb. comp.: **NLOGSPACE**
d.c. instance: in **$AC^0$**
d.c. query: **coNP**

TBox axioms $B_1 \sqsubseteq B_2 \quad B_1 \sqsubseteq \neg B_2 \quad \neg B_1 \sqsubseteq B_2$

4. $\mathit{DL\text{-}Lite}^{\mathcal{N}}_{core} = \mathit{DL\text{-}Lite}^{\mathcal{N}}_{horn} \cap \mathit{DL\text{-}Lite}^{\mathcal{N}}_{krom}$

comb. comp.: **NLOGSPACE**
d.c. instance: in **$AC^0$**
d.c. query: in **$AC^0$**

**NB:** complexity by embedding in the 1-variable fragments of FOL

**NB:** in $AC^0$ informally means 'as effective as relational databases', i.e., FO-rewritable

**NB:** **UNA** is essential for encoding number restrictions

# Embedding *DL-Lite* into FOL

**Theorem** The satisfiability problem for *DL-Lite*$^{\mathcal{N}}_{bool}$ knowledge bases is **NP**-complete

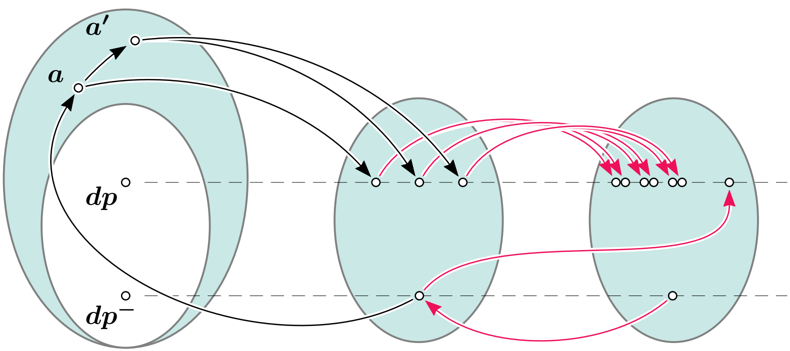
Proof by embedding into the 1-variable fragment of first-order logic

$$\mathcal{T} = \{A \sqsubseteq \exists P^-,\ \exists P^- \sqsubseteq A,\ A \sqsubseteq\ \geq 2\, P,\ \top \sqsubseteq\ \leq 1\, P^-,\ \exists P \sqsubseteq A\},\ \mathcal{A} = \{A(a), P(a,a')\}$$

$$\forall x \Big[ (A(x) \to E_1P^-(x)) \land (E_1P^-(x) \to A(x)) \land (A(x) \to E_2P(x)) \land \neg E_2P^-(x) \land (E_1P(x) \to A(x)) \land (E_2P(x) \to E_1P(x)) \land (E_2P^-(x) \to E_1P^-(x)) \land (E_1P(x) \to E_1P^-(dp^-)) \land (E_1P^-(x) \to E_1P(dp)) \Big] \land A(a) \land E_1P(a) \land E_1P^-(a')$$

$(\exists P)^{\mathcal{I}} \neq \emptyset$ iff $(\exists P^-)^{\mathcal{I}} \neq \emptyset$
$(\exists x\, E_1P(x) \leftrightarrow \exists x\, E_1P^-(x))$

all points are in $A, \exists P^-, \geq 2\, P$




No fmp,
but only **linear** number of
(domain and range)
witnesses needed **!**

# The *DL-Lite* family revisited

**3D** family $\textit{DL-Lite}_{\alpha}^{\beta,\gamma}$

$\alpha$ concept inclusions

*core*: $B_1 \sqsubseteq B_2, \; B_1 \sqsubseteq \neg B_2$

*Krom*: $B_1 \sqsubseteq B_2, \; B_1 \sqsubseteq \neg B_2, \; \neg B_1 \sqsubseteq B_2$

*Horn*: $B_1 \sqcap \cdots \sqcap B_k \sqsubseteq B$

*Bool*: $C_1 \sqsubseteq C_2$

$\beta$ role inclusions

$\mathcal{R}$: $R_1 \sqsubseteq R_2$

: not allowed

$\gamma$ number restrictions

: only $\exists R$

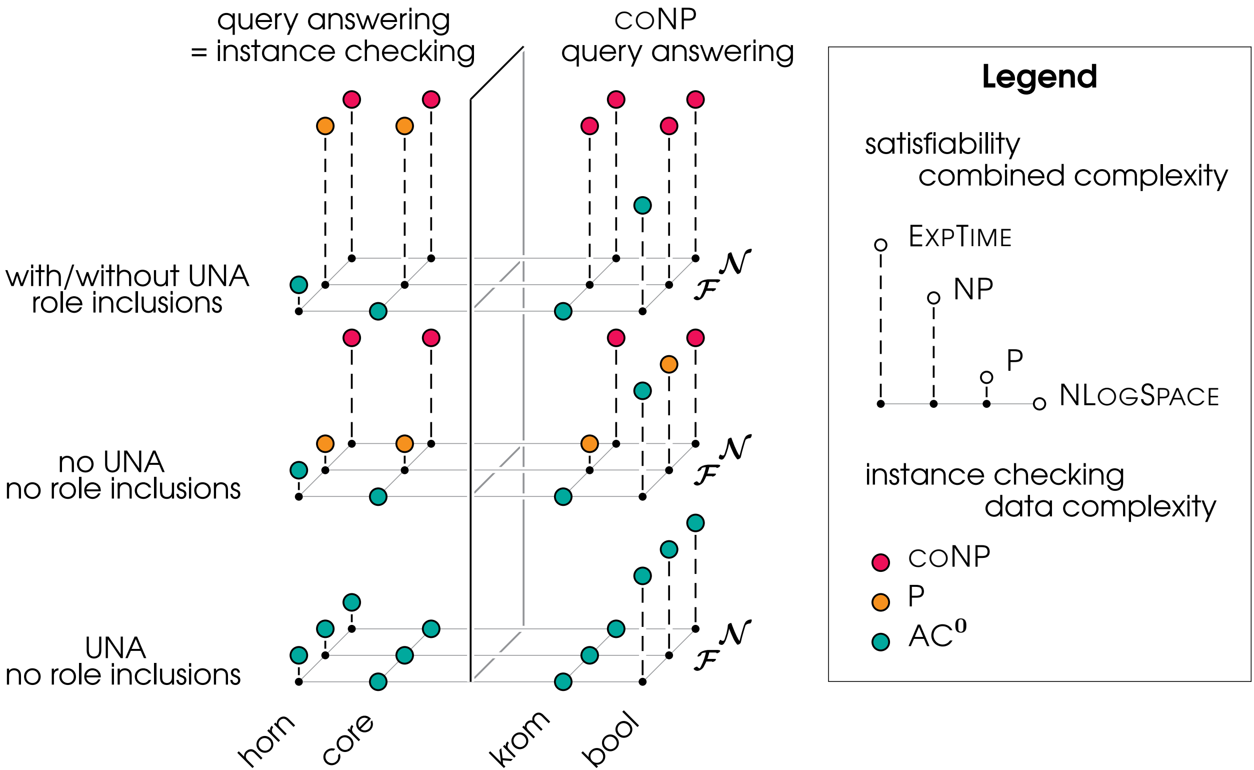
$\mathcal{F}$: $\exists R$ and $\geq 2\, R \sqsubseteq \bot$ (functionality constraints)

$\mathcal{N}$: $\geq q\, R$

**+ 4th dimension**: unique name assumption

| |
|---|
| **UNA:** $a_i^{\mathcal{I}} \neq a_j^{\mathcal{I}}$ for all $i \neq j$ |

# The *DL-Lite* family: complexity-scape



# Unique Name Assumption (UNA)

**Theorem** The satisfiability problem for *DL-Lite*$^{\mathcal{F}}_{core}$ (functionality only) KBs is **P**-complete for data complexity

<u>Proof</u> $\varphi = \bigwedge_{k=1}^{n} (a_{k,1} \land a_{k,2} \to a_{k,3}) \land \bigwedge_{l=1}^{p} a_{l,0}$ (a Horn-3CNF formula)

each $a_{k,j}$ and each $a_{l,0}$ is one of the propositional variables $a_1, \dots, a_m$

$a_{k,1}, a_{k,2}, a_{k,3}$ are all distinct.

P-complete problem '$\varphi \models a_j$?'

$\mathcal{T}$: $\geq 2\, P \sqsubseteq \bot, \quad \geq 2\, Q \sqsubseteq \bot, \quad \geq 2\, S \sqsubseteq \bot, \quad \geq 2\, T \sqsubseteq \bot$

object names $t$, $a_i^k$, $f_k$, $g_k$, for $1 \leq k \leq n$, $1 \leq i \leq m$

$\mathcal{A}$: $S(a_i^1, a_i^2), \quad \dots, \quad S(a_i^{n-1}, a_i^n), \quad S(a_i^n, a_i^1)$, for $1 \leq i \leq m$

$P(a_{k,1}^k, f_k), \quad P(a_{k,2}^k, g_k), \quad Q(g_k, a_{k,3}^k), \quad Q(f_k, a_{k,1}^k)$, for $1 \leq k \leq n$

$T(t, a_{l,0}^1)$, for $1 \leq l \leq p$

$\varphi \models a_j$ iff $(\mathcal{T}, \mathcal{A}) \models T(t, a_j^1)$ (without the UNA)

**NB:** P-completeness means that it is not FO-rewritable (in fact, it's FO + LFP)

# FO-rewritability = in $AC^0$ (rather than in LogSpace)

**FO-rewritability**:

given a query $q(\vec{a})$ and a TBox $\mathcal{T}$ one can construct a query $q_{\mathcal{T}}(\vec{x})$ such that

$$(\mathcal{T}, \mathcal{A}) \models q(\vec{a}) \qquad \text{iff} \qquad \mathfrak{A}_{\mathcal{A}} \models q_{\mathcal{T}}(\vec{a}),$$

where $\mathfrak{A}_{\mathcal{A}}$ is *the* first-order model induced by $\mathcal{A}$

**Fact** Model checking in FOL (evaluating a FO-formula) is in $AC^0$ for data complexity

**circuit** = DAG built from unbounded fan-in And, Or and Not gates

> $AC^0$ is the class of problems definable using
> a family of circuits of **constant depth** and **polynomial size**,
> which can be generated by a deterministic Turing machine in logarithmic time
> (in the size of the input) LogTime-uniformity

i.e., $AC^0$ stands for polynomially many processors with the constant run-time

**NB:** Parity is in LogSpace but not in $AC^0$

> (Immerman 1989, Dawar *et al* 1998) $AC^0$ = FO + BIT(x,y)

**Theorem** Without the UNA, instance checking in *DL-Lite*$_{core}$ with equalities ($a \approx b$) is **LogSpace-complete** for data complexity (in particular, **not FO-rewritable**)

# Delicate balance: either numbers restrictions or role inclusions

$DL\text{-}Lite^{\mathcal{F}}_{core}$ (i.e., $B_1 \sqsubseteq B_2$, $B_1 \sqsubseteq \neg B_2$) is **NLogSpace**-complete for combined complexity and in **$AC^0$** for data complexity (under the UNA)

$DL\text{-}Lite^{\mathcal{R},\mathcal{F}}_{core}$ ($DL\text{-}Lite^{\mathcal{F}}_{core}$ + $R_1 \sqsubseteq R_2$) is **ExpTime**-complete for combined complexity and **P**-complete for data complexity

**Example 1:** $A_1 \sqcap A_2 \sqsubseteq C$ can be simulated by the axioms:

$$
\begin{array}{rclcrcl}
A_1 & \sqsubseteq & \exists R_1 & \quad & A_2 & \sqsubseteq & \exists R_2 \\
R_1 & \sqsubseteq & R_{12} & & R_2 & \sqsubseteq & R_{12} \\
\geq 2\, R_{12} & \sqsubseteq & \bot & & & & \\
\exists R_1^- & \sqsubseteq & \exists R_3^- & & & & \\
\exists R_3 & \sqsubseteq & C & & & & \\
R_3 & \sqsubseteq & R_{23} & & R_2 & \sqsubseteq & R_{23} \\
\geq 2\, R_{23}^- & \sqsubseteq & \bot & & & &
\end{array}
$$

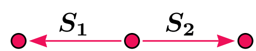
# Delicate balance: either numbers restrictions or role inclusions (2)

$DL\text{-}Lite_{core}^{\mathcal{R},\mathcal{F}}$ $(DL\text{-}Lite_{core}^{\mathcal{F}}$ + $R_1 \sqsubseteq R_2)$ is **ExpTime**-complete for combined complexity and **P**-complete for data complexity

**Example 2:** $A \sqsubseteq \exists R.B$ can be simulated by the axioms:

$$A \sqsubseteq \exists R_B \qquad R_B \sqsubseteq R \qquad \exists R_B^- \sqsubseteq C$$

**Example 3:** $A \sqsubseteq \forall R.B$ can be simulated by using **reification**:


$$\geq 2\, S_k \sqsubseteq \bot, \qquad S_{k,B} \sqsubseteq S_k \quad \text{and} \quad S_{k,\neg B} \sqsubseteq S_k, \qquad \text{for } k = 1, 2$$

$$\exists S_{1,B} \equiv \exists S_{2,B} \quad \text{and} \quad \exists S_{1,\neg B} \equiv \exists S_{2,\neg B}$$

$$\exists S_2 \sqsubseteq \exists S_{2,B} \sqcup \exists S_{2,\neg B}$$

$$\exists S_{2,B}^- \sqsubseteq B \quad \text{and} \quad \exists S_{2,\neg B}^- \sqsubseteq \neg B$$

$$A \sqsubseteq \neg \exists S_{1,\neg B}^-$$

# *DL-Lite*$_\alpha^{(\mathcal{RN})}$: pushing the limits of *DL-Lite*

- **role inclusions + number restrictions** (like in *DL-Lite*$_\mathcal{A}$)

if $R$ has a proper sub-role in $\mathcal{T}$ then $\mathcal{T}$ contains
no *negative occurrences* of $\geq q\, R$ or $\geq q$ *inv*$(R)$ with $q \geq 2$

- positive occurrences of qualified number restrictions $\geq q\, R.C$

if $\geq q\, R.C$ occurs in $\mathcal{T}$ then $\mathcal{T}$ contains
no *negative occurrences* of $\geq q'\, R$ or $\geq q'$ *inv*$(R)$ with $q' \geq 2$

no TBox can contain both a functionality constraint $\geq 2\, R \sqsubseteq \bot$ and $\geq q\, R.C$, for any $q \geq 1$

- role **disjointness**, **symmetry**, **asymmetry**, **reflexivity** and **irreflexivity** constraints

**all these extensions do not change the complexity**
**in particular, *DL-Lite*$_\alpha^{(\mathcal{RN})}$ is the same as *DL-Lite*$_\alpha^{\mathcal{N}}$**

**NB.** transitive roles do not change the combined complexity
(NLOGSPACE-hard for data complexity)

# Publications

[1] A. Artale, D. Calvanese, R. Kontchakov and M. Zakharyaschev.
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School of Computer Science and Information Systems, Birkbeck, University of London.

[2] R. Kontchakov and M. Zakharyaschev. *DL-Lite and role inclusions*.
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pp. 16–30, LNCS, vol. 5367, Springer 2008.

[3] A. Artale, D. Calvanese, R. Kontchakov, V. Ryzhikov and M. Zakharyaschev.
*Reasoning over Extended ER models*.
In C. Parent, K.-D. Schewe and V. C. Storey, editors, ER 2007 Conference Proceedings (Auckland, New Zealand, November 5–9, 2007), LNCS vol. 4801,
pp. 277–292. Springer, 2007.

[4] A. Artale, D. Calvanese, R. Kontchakov and M. Zakharyaschev.
*DL-Lite in the Light of First-Order Logic*.
In Proceedings of the 22nd AAAI Conference on Artificial Intelligence
(Vancouver, British Columbia, July 22-26, 2007), pp. 361–366. AAAI Press, 2007.