Temporalising tractable description logics

Roman Kontchakov

School of Computer Science and Information Systems, Birkbeck, London

http://www.dcs.bbk.ac.uk/~roman

joint work with

Alessandro Artale, Carsten Lutz, Frank Wolter and Michael Zakharyaschev
Temporalising description logics

- Description Logics (DLs) have proven to be adequate for modelling (ontologies, OWL, databases, ...)

- So far, attempts to construct decidable temporal extensions of DLs have been unsuccessful (constructed logics are either inexpressive or undecidable)

- However, recently new families of (tractable) DLs have been identified (DL-Lite and EL)

We investigate temporalisations of these tractable DLs (with hope that they are decidable)
Description logics

- **concepts** (sets of objects)
  \[
  C ::= \bot \mid A \mid \neg C \mid C_1 \cap C_2 \mid \exists R.C \mid \forall R.C \mid \leq q.C \mid \geq q.C
  \]

- **roles** (binary relations between objects)
  \[
  R ::= P \mid P^-
  \]

- **TBox axioms and ABox assertions** (propositions)
  \[
  \begin{align*}
  &C_1 \sqsubseteq C_2, \quad C(a), \quad R(a_1, a_2) \\
  &| \quad \text{TBox} \quad \text{ABox}
  \end{align*}
  \]

  \[
  \mathcal{T} = \{ \text{Feline} \sqsubseteq \forall \text{eats. Animal}, \quad \text{Cat} \sqsubseteq \text{Feline} \}, \quad \mathcal{A} = \{ \text{Cat(tom)} \}
  \]
Temporal description logics

- **concepts**
  - $\Diamond C$ ‘in $C$ tomorrow’
  - $\Box F C$ ‘always in $C$’
  - $\Diamond F C$ ‘eventually in $C$’

- **roles**
  

\[
R ::= P \mid P^- \mid T \mid T^-
\]

  local (may change) \hspace{1cm} global (stay constant)

- **formulas**

\[
\varphi ::= C_1 \sqsubseteq C_2 \mid C(a) \mid R(a_1, a_2) \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \Diamond \varphi \mid \Box F \varphi \mid \Diamond F \varphi
\]

**Theorem** The satisfiability problem for $\mathcal{T L}_{\mathcal{ALC}}$-formulas

1. **without global roles** is $\text{ExpSpace}$-complete
2. with a single **global role** is **undecidable**

**NB:** a global role allows one to model, say, the tape of a Turing machine
Capturing Entity-Relationship diagrams in DLs

Translating into a description logic:

\[ \exists \text{passengers}. \top \sqsubseteq \text{Flight} \]
\[ \exists \text{passengers}^-. \top \sqsubseteq \text{Passenger} \]

\[ \top \geq q \ R. \top \text{ is enough for ER!} \]

\[ \exists R. \top \equiv \geq 1 \ R. \top \]

\begin{align*}
\text{Flight} & \sqsubseteq \geq 2 \text{crew-members}. \top \\
\text{CabinCrew} & \sqsubseteq \text{Person} \\
\text{CabinCrew} & \sqsubseteq \text{Pilot} \sqcup \text{Steward} \\
\text{Pilot} & \sqcap \text{Steward} \sqsubseteq \bot
\end{align*}
DL-Lite and its sublanguages

1. **DL-Lite\text{bool}** (captures full ER) \[ B ::= \bot \mid A \mid \geq qR \]
   \[ C ::= B \mid \neg C \mid C_1 \sqcap C_2 \]
   TBox axioms \[ C_1 \sqsubseteq C_2 \]
   \text{NP-complete}

2. **DL-Lite\text{horn}**
   TBox axioms \[ B_1 \sqcap \cdots \sqcap B_n \sqsubseteq B \]
   \text{P-complete}

3. **DL-Lite\text{krom}** (ER without covering constraints, e.g., \( B \sqsubseteq B_1 \sqcup B_2 \))
   TBox axioms \[ B_1 \sqsubseteq B_2 \quad B_1 \sqsubseteq \neg B_2 \quad \neg B_1 \sqsubseteq B_2 \]
   (subclass) (disjointness)
   \text{NLogSpace-complete}

\textbf{NB:} these complexity results are closely connected to the complexity of reasoning in fragments of propositional logic
**Theorem** The satisfiability problem for TDL-Lite\_bool-formulas is **ExpSPACE**-complete (with or without global roles)

upper bound: embedding into the one-variable fragment of first-order temporal logic (FOTL)

lower bound: converse embedding

\[
\forall x \psi(x) \leadsto T \subseteq \psi^*
\]
Temporal **DL-Lite**\textsubscript{krom}

- **concepts**
  \[ C ::= \bot \mid A \mid \geq qR \mid \neg C \mid \Diamond C \]

- **formulas**
  \[ \varphi ::= C_1 \sqsubseteq C_2 \mid C(a) \mid R(a_1, a_2) \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \Diamond \varphi \mid \Box_F \varphi \mid \Diamond_F \varphi \]

**NB:** many types of temporal constraints can be defined in **TDL-Lite**\textsubscript{krom} (except covering and temporary entities/relations)

**Theorem** The satisfiability problem for **TDL-Lite**\textsubscript{krom}-formulas is **PSpace**-complete

**upper bound:** `saturated’ quasimodels (see also Balbiani&Condotta 2002, Lutz&Miličić 2005)

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**Temporal **DL-Lite**\textsubscript{horn}

**Theorem** The satisfiability problem for **TDL-Lite**\textsubscript{horn}-formulas is **ExpSpace**-complete

**lower bound:** \(2^n\)-corridor tiling (modification of the proof for the one-variable fragment of FOTL)
\[ \mathcal{EL}: \text{another tractable DL} \]

- concepts
  \[ C ::= \top \mid A \mid C_1 \sqcap C_2 \mid \exists P.C \]  
  \( (P \text{ is a role name}) \)

- general concept inclusions (CGIs)
  \[ C_1 \sqsubseteq C_2 \]

\( \mathcal{EL} \) is largely motivated by applications (SNOMED, Galen, GO, \ldots)

**NB:** every TBox is satisfiable, so the main inference problem is subsumption:

given a TBox (a set of GCIs) \( T \) and a GCI \( C_1 \sqsubseteq C_2 \), decide
whether \( C_1 \sqsubseteq C_2 \) holds in every model for \( T \)

(\textit{Baader 2003, Brandt 2004}): The subsumption problem for \( \mathcal{EL} \) is in \( P \)
Temporal \( \mathcal{EL} \)

- concepts

\[
C ::= \top | A | C_1 \cap C_2 | \exists R.C | \Box_F C | \ldots
\]

\( (R \) is a local or global role name)\)

**Theorem** The satisfiability problem for \( \mathcal{TL}_{\mathcal{EL}} \)-formulas is **undecidable**

more precisely,

it is undecidable whether a \( \mathcal{TL}_{\mathcal{EL}} \) GCI is a consequence of a set of \( \mathcal{TL}_{\mathcal{EL}} \) GCIs

here, GCIs are of the form \( \Box_F^+(C_1 \sqsubseteq C_2) \)

the proof is by encoding \( \mathcal{TL}_{\mathcal{ALC}} \) GCIs in \( \mathcal{TL}_{\mathcal{EL}} \) (\( \Box \) is the only difficult case):

- \( \Box_F^+(C \sqsubseteq \exists R.(M \sqcap \diamond_F X \sqcap \diamond_F Y)) \) both \( X \) and \( Y \) will happen and either
- \( \Box_F^+(\exists R.(M \sqcap \diamond_F X \sqcap \diamond_F Y) \sqsubseteq A) \) \( X \) is before \( Y \)
- \( \Box_F^+(\exists R.(M \sqcap \diamond_F Y \sqcap \diamond_F X) \sqsubseteq A) \) \( Y \) is before \( X \)
- \( \Box_F^+(\exists R.(M \sqcap \diamond_F (X \sqcap Y)) \sqsubseteq B) \) or \( X \) and \( Y \) are at the same time
Conclusions

- Absence of ‘qualified’ quantification makes temporalisations of DL-Lite decidable (in fact, these logics are very similar to the one-variable fragment of FOTL)

- Role inclusions in DL-Lite and qualified quantification in EL ruin decidability of temporalisations