Temporalising tractable description logics

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Temporalising description logics

- Description Logics (DLs) have proven to be adequate for modelling

  (ontologies, OWL, databases, ...)

- So far, attempts to construct decidable temporal extensions of DLs

  have been unsuccessful

  (constructed logics are either inexpressive or undecidable)

- However, recently new families of (tractable) DLs

  have been identified (*DL-Lite* and $\mathcal{EL}$)

We investigate temporalisations of these tractable DLs

(with hope that they are decidable)

Description logics

- concepts (sets of objects)

$$C \;::=\; \bot \;\mid\; A \;\mid\; \neg C \;\mid\; C_1 \sqcap C_2 \;\mid\; \exists R.C \;\mid\; \forall R.C \;\mid\; \leq q.C \;\mid\; \geq q.C$$

$\exists \mathit{eats}.\mathit{Animal}$, ('carnivores')

$\forall \mathit{eats}.\mathit{Plant}$, ('herbivores')

$\mathit{Animal} \sqcap \geq 3\, \mathit{offspring}.\top$ ('animals with at least three offspring')

- roles (binary relations between objects)

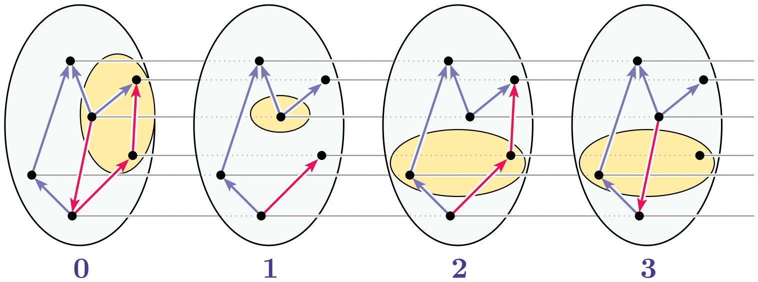
$$R \;::=\; P \;\mid\; P^-$$

- TBox axioms and ABox assertions (propositions)

$$\underbrace{C_1 \sqsubseteq C_2,}_{\text{TBox}} \qquad \underbrace{C(a), \qquad R(a_1, a_2)}_{\text{ABox}}$$

$$\mathcal{T} = \{\ \mathit{Feline} \sqsubseteq \forall \mathit{eats}.\mathit{Animal}, \quad \mathit{Cat} \sqsubseteq \mathit{Feline}\ \}, \qquad \mathcal{A} = \{\ \mathit{Cat}(\mathit{tom})\ \}$$

Temporal description logics



- concepts

$\bigcirc C$	'in C tomorrow'
$\Box_F C$	'always in C'
$\Diamond_F C$	'eventually in C'

- roles

$$R \; ::= \; P \mid P^- \;\big|\; T \mid T^-$$

local (may change) | global (stay constant)

- formulas

$$\varphi \; ::= \; C_1 \sqsubseteq C_2 \mid C(a) \mid R(a_1, a_2) \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \bigcirc\varphi \mid \Box_F\varphi \mid \Diamond_F\varphi$$

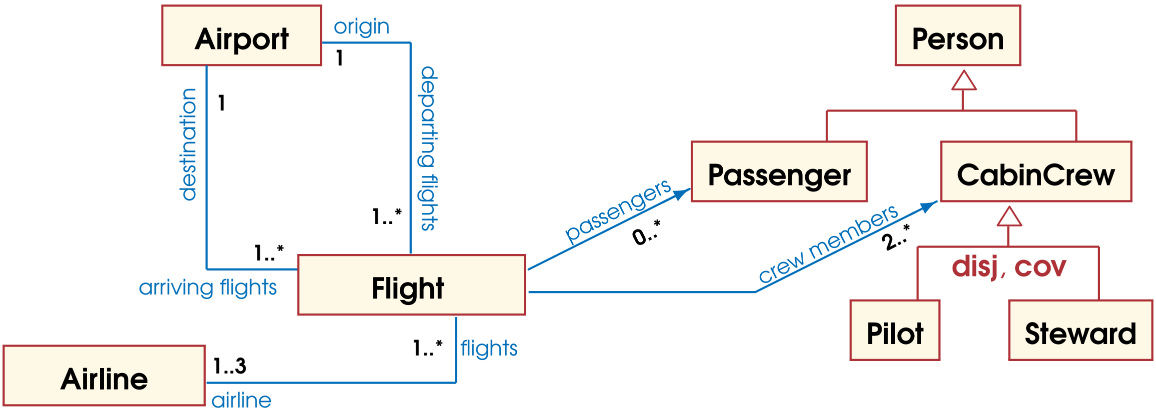
Theorem The satisfiability problem for $\mathcal{TL}_{\mathcal{ALC}}$-formulas

(1) **without global roles** is **ExpSpace**-complete

(2) with a single **global role** is **undecidable**

NB: a global role allows one to model, say, the tape of a Turing machine

Capturing Entity-Relationship diagrams in DLs



Translating into a description logic:

$$\exists\,\text{passengers}.\top \sqsubseteq \text{Flight}$$

$$\exists\,\text{passengers}^{-}.\top \sqsubseteq \text{Passenger}$$

$$\text{Flight} \sqsubseteq\ \geq 2\ \text{crew-members}.\top$$

$$\text{CabinCrew} \sqsubseteq \text{Person}$$

$$\text{CabinCrew} \sqsubseteq \text{Pilot} \sqcup \text{Steward}$$

$$\text{Pilot} \sqcap \text{Steward} \sqsubseteq \bot$$

$\geq q\ R.\top$ is enough for ER! $\quad \exists R.\top \equiv\ \geq 1\ R.\top$

DL-Lite and its sublanguages

1. *DL-Lite$_{bool}$* (captures full ER) **NP**-complete

$$B \ ::= \ \bot \ \mid \ A \ \mid \ \geq qR$$
$$C \ ::= \ B \ \mid \ \neg C \ \mid \ C_1 \sqcap C_2$$

TBox axioms $C_1 \sqsubseteq C_2$

2. *DL-Lite$_{horn}$* **P**-complete

TBox axioms $B_1 \sqcap \cdots \sqcap B_n \sqsubseteq B$

3. *DL-Lite$_{krom}$* (ER without covering constraints, e.g., $B \sqsubseteq B_1 \sqcup B_2$) **NLogSpace**-complete

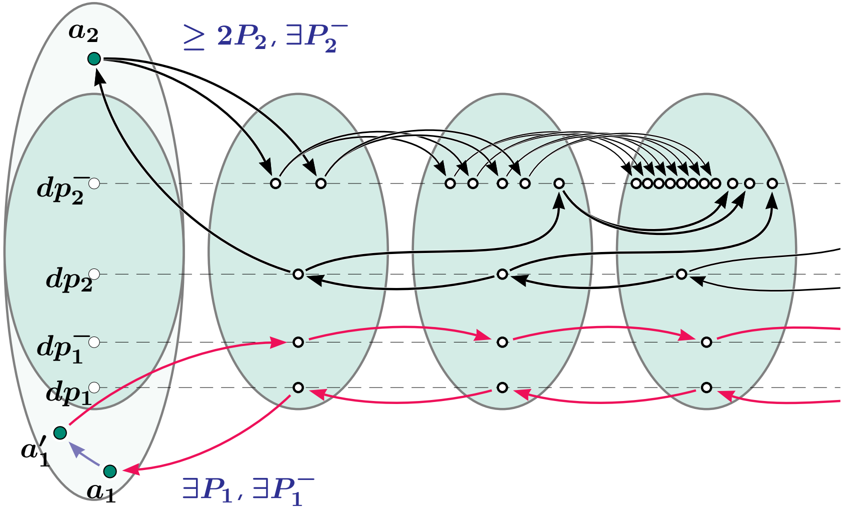
TBox axioms $B_1 \sqsubseteq B_2$ (subclass) $B_1 \sqsubseteq \neg B_2$ (disjointness) $\neg B_1 \sqsubseteq B_2$

NB: these complexity results are closely connected to the complexity of reasoning in fragments of propositional logic

Temporal *DL-Lite*$_{bool}$

Theorem The satisfiability problem for *TDL-Lite*$_{bool}$-formulas is **ExpSpace**-complete
(with or without global roles)

upper bound: embedding into the one-variable fragment
of first-order temporal logic (FOTL)



$(\exists P)^{\mathcal{I}} \neq \emptyset$
iff
$(\exists P^-)^{\mathcal{I}} \neq \emptyset$

lower bound: converse embedding $\quad \forall x\, \psi(x) \quad \rightsquigarrow \quad \top \sqsubseteq \psi^*$

Temporal *DL-Lite*$_{krom}$

- concepts $C \quad ::= \quad \bot \quad | \quad A \quad | \quad \geq qR \quad | \quad \neg C \quad | \quad \bigcirc C$
- formulas $\varphi \quad ::= \quad C_1 \sqsubseteq C_2 \quad | \quad C(a) \quad | \quad R(a_1, a_2) \quad | \quad \neg\varphi \quad | \quad \varphi_1 \wedge \varphi_2 \quad | \quad \bigcirc\varphi \quad | \quad \Box_F\varphi \quad | \quad \Diamond_F\varphi$

NB: many types of temporal constraints can be defined in *TDL-Lite*$_{krom}$

(except covering and temporary entities/ralations)

Theorem The satisfiability problem for *TDL-Lite*$_{krom}$-formulas is **PSPACE**-complete

upper bound: 'saturated' quasimodels (see also Balbiani&Condotta 2002, Lutz&Miličić 2005)

Temporal *DL-Lite*$_{horn}$

Theorem The satisfiability problem for *TDL-Lite*$_{horn}$-formulas is **EXPSPACE**-complete

lower bound: 2^n-corridor tiling (modification of the proof for the one-variable fragment of FOTL)

$\mathcal{EL}$: another tractable DL

- concepts $\quad C \; ::= \; \top \;\mid\; A \;\mid\; C_1 \sqcap C_2 \;\mid\; \exists P.C$

(P is a role name)

- general concept inclusions (CGIs) $\quad C_1 \sqsubseteq C_2$

$\mathcal{EL}$ is largely motivated by applications (SNOMED, Galen, GO, ...)

NB: every TBox is satisfiable, so the main inference problem is <u>subsumption</u>:

given a TBox (a set of GCIs) $\mathcal{T}$ and a GCI $C_1 \sqsubseteq C_2$, decide whether $C_1 \sqsubseteq C_2$ holds in every model for $\mathcal{T}$

(Baader 2003, Brandt 2004): The subsumption problem for $\mathcal{EL}$ is in **P**

Temporal $\mathcal{EL}$

- concepts $C \; ::= \; \top \;|\; A \;|\; C_1 \sqcap C_2 \;|\; \exists R.C \;|\; \Diamond_F C \;|\; \dots$

(R is a local or global role name)

Theorem The satisfiability problem for $\mathcal{TL}_{\mathcal{EL}}$-formulas is **undecidable**

more precisely,

it is undecidable whether a $\mathcal{TL}_{\mathcal{EL}}$ GCI is a consequence of a set of $\mathcal{TL}_{\mathcal{EL}}$ GCIs

here, GCIs are of the form $\Box_F^+(C_1 \sqsubseteq C_2)$

the proof is by encoding $\mathcal{TL}_{\mathcal{ALC}}$ CGIs in $\mathcal{TL}_{\mathcal{EL}}$ ($\sqcup$ is the only difficult case):

$\Box_F^+(C \sqsubseteq \exists R.(M \sqcap \Diamond_F X \sqcap \Diamond_F Y))$ both X and Y will happen

and either

$\Box_F^+(C \sqsubseteq A \sqcup B) \quad \rightsquigarrow \quad \Box_F^+(\exists R.(M \sqcap \Diamond_F(X \sqcap \Diamond_F Y)) \sqsubseteq A)$ X is before Y

$\Box_F^+(\exists R.(M \sqcap \Diamond_F(Y \sqcap \Diamond_F X)) \sqsubseteq A)$ Y is before X

$\Box_F^+(\exists R.(M \sqcap \Diamond_F(X \sqcap Y)) \sqsubseteq B)$ or X and Y are at the same time

Conclusions

- Absence of `qualified' quantification makes
  temporalisations of *DL-Lite* decidable

  (in fact, these logics are very similar to the one-variable fragment of FOTL)

- Role inclusions in *DL-Lite* and qualified quantification in $\mathcal{EL}$
  ruin decidability of temporalisations