Birkbeck
(University of London)

MSc and MRes Examination for Internal Students
MSc in Advanced Information Systems
MRes in Computer Science

School of Computer Science and Information Systems

Knowledge Representation and Reasoning (COIY027P)

Date of examination: 31/05/2005
Duration of paper: (14:30–16:30)

There are six questions on this paper. Candidates should attempt any FOUR of them. Calculators are not permitted.

1. Consider the following formula φ: $(\Diamond p \to q) \leftrightarrow (\neg q \to \Box\neg p)$.

(a) Draw the parse tree for φ. (5 marks)

(b) List all the subformulas of φ. (5 marks)

(c) Rewrite φ in an equivalent form by using the connectives $\neg$, $\wedge$ and $\Diamond$ only. (10 marks)

(d) Determine whether φ is valid. (5 marks)

2. A frame is called *partially functional* if, for every possible world, there is at most one possible world it is related to, i.e.,

$$\forall s \forall t \forall u[(sRt \wedge sRu) \to t = u].$$

Consider the following formula

$$\text{PF:} \qquad \Diamond\varphi \to \Box\varphi.$$

(a) Prove that the formula PF is valid in partially functional frames. (15 marks)

(b) Show that the formula PF is not valid in general. (10 marks)

3. Consider the following variant of the muddy children puzzle. There are four children and the father's public announcement is that

"there are more muddy children than clean children".

(a) Draw a Kripke frame representing the children's knowledge after the father has spoken. (5 marks)

(b) Using epistemic logic, determine how many rounds are needed for the children to find out if they are muddy. (10 marks)

(c) How does the situation change if one of the children is deaf (hence unaware of the father's announcement, but aware of the rules of the game)? (10 marks)

4. (a) Determine whether the following formula of linear temporal logic is valid:

$$\varphi \mathsf{U} (\psi \vee \chi) \leftrightarrow (\varphi \mathsf{U} \psi \vee \varphi \mathsf{U} \chi).$$

(10 marks)

(b) Write a linear temporal logic formula expressing the following specification:

"the event φ will not occur before event ψ, which (i.e., ψ) may or may not occur at all".

(10 marks)

(c) Formulate the basic invariance rule for program specification. (5 marks)

5. Consider the following concurrent program.

input variables: $y_1 = 0$ $y_2 = 0$

$$P_1 \left[\begin{array}{l} l_0 : \textbf{while}(\text{true})\{ \\ l_1 : \textbf{noncritical} \\ l_2 : y_1 = 1 \\ l_3 : \textbf{await}(y_2 = 0) \\ l_4 : \textbf{critical} \\ l_5 : y_1 = 0\} \end{array}\right] \quad || \quad P_2 \left[\begin{array}{l} m_0 : \textbf{while}(\text{true})\{ \\ m_1 : \textbf{noncritical} \\ m_2 : y_2 = 1 \\ m_3 : \textbf{await}(y_1 = 0) \\ m_4 : \textbf{critical} \\ m_5 : y_2 = 0\} \end{array}\right]$$

(a) Determine whether the following formula is inductive:

$$(at\_l_3 \wedge at\_m_3) \leftrightarrow (y_1 = 1 \wedge y_2 = 1)$$

(5 marks)

(b) Show that the following formula is invariant:

$$(at\_l_3 \wedge at\_m_3) \rightarrow \mathsf{G}(at\_l_3 \wedge at\_m_3)$$

[Hint: use assertion strengthening.] (20 marks)

6. Consider the follwing definition: $\Box^1\varphi = \Box\varphi$ and $\Box^{n+1}\varphi = \Box(\Box^n\varphi)$ for any natural number n. Show that, for any model $\mathcal{M} = (W, R, v)$ of modal logic and world $s \in W$,

$$(\mathcal{M}, s) \models \Box^n\varphi$$
if and only if
$(\mathcal{M}, t) \models \varphi$ for all t that are R-reachable from s in n steps.

(25 marks)

