Birkbeck
(University of London)

MSc Examination

Department of Computer Science and Information Systems

FUNDAMENTALS OF CONCURRENT SYSTEMS (formerly KNOWLEDGE REPRESENTATION AND REASONING) (COIY027H7)

CREDIT VALUE: 15 credits

Date of examination: 27/05/2014
Duration of paper: 14:30–16:30

There are six questions in this paper, each worth 25 marks.
Candidates who are taking the module for the first time should answer questions 5 and 6 and any two additional questions.
Resit candidates should answer any four questions.
Calculators are not permitted.

1. (a) Consider the following argument.

"If I am clever, then I pass the exam, unless I am unlucky. Therefore, if I am both clever and lucky, then I pass the exam."

Formalise the above argument in propositional logic and determine whether it is valid. (12 marks)

(b) Prove that the following semantical implication is correct for any natural number n:

$$(a_1 \to (a_2 \to (a_3 \to (\ldots \to (a_{n-1} \to (a_n \to b))\ldots)))) \models$$
$$(a_1 \wedge a_2 \wedge a_3 \wedge \ldots \wedge a_{n-1} \wedge a_n) \to b$$

[Hint: Use induction on n.] (13 marks)

2. A frame $\mathcal{F} = (W, R)$ is called *directed* if

$$\forall s \forall t \forall u((sRt \wedge sRu) \to \exists v(tRv \wedge uRv))$$

as illustrated by the picture:

$$\begin{tikzcd}
& t \arrow[dr, dashed, "R"] & \\
s \arrow[ur, "R"] \arrow[dr, "R"'] & & v \\
& u \arrow[ur, dashed, "R"'] &
\end{tikzcd}$$

Consider the following formula:

$$\text{DIRECTED:} \qquad \Diamond\Box\varphi \to \Box\Diamond\varphi$$

(a) Prove that the formula DIRECTED is valid in directed frames. (15 marks)

(b) Show that the formula DIRECTED is not valid in general. (10 marks)

3. We define $\Diamond^1\varphi = \Diamond\varphi$ and $\Diamond^{n+1}\varphi = \Diamond(\Diamond^n\varphi)$ for any natural number n.

(a) Show that, for every model $\mathcal{M} = (W, R, v)$ of modal logic and every world $s \in W$, the following claim holds:

$(\mathcal{M}, s) \models \Diamond^n\varphi$
if and only if
$(\mathcal{M}, t) \models \varphi$ for some t that is R-reachable from s in n steps.

(15 marks)

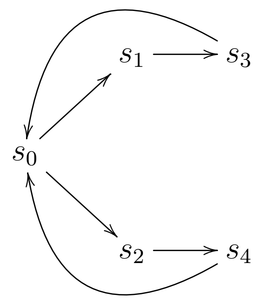
(b) Explain how the above claim can be simplified for epistemic logic. (10 marks)

4. Consider the temporal logic formulas

$$\varphi_1 : \ (\alpha \vee \beta) U \gamma \qquad \varphi_2 : \ (\alpha U \gamma) \vee (\beta U \gamma)$$

(a) Determine whether $\varphi_1 \to \varphi_2$ and $\varphi_2 \to \varphi_1$ are valid linear temporal logic formulas. (15 marks)

(b) Determine whether $\varphi_1 \wedge \varphi_2$ is valid over the following labelled transition system $\mathcal{T}$. There are five states: s_0 (the initial state), s_1, s_2, s_3 and s_4. The transitions are $\{(s_0, s_1), (s_0, s_2), (s_1, s_3), (s_2, s_4), (s_3, s_0), (s_4, s_0)\}$ as illustrated by the picture below:



The labelling is given by $\ell(s_0) = \{\alpha, \beta\}$, $\ell(s_1) = \{\alpha\}$, $\ell(s_2) = \{\beta\}$ and $\ell(s_3) = \ell(s_4) = \{\gamma\}$. (10 marks)

5. Consider the following attempt for the *dining philosophers* problem.

```
semaphore s = 1            /* binary semaphore */
semaphore fork[5] = 1      /* binary semaphores for 0 to 4 */
int i          /* philosopher number from 0 to 4 */

void philosopher(int i)
{
L0 : while(true){
L1 :  think();
L2 :  request(s);
L3 :  request(fork[i]);
L4 :  request(fork[i + 1]  mod 5);
L5 :  signal(s);
L6 :  eat();
L7 :  signal(fork[i]);
L8 :  signal(fork[i + 1]  mod 5);
 }
}
```


(a) Briefly explain how to create a labelled transition system $\mathcal{T}$ associated to this program. (You do not have to draw $\mathcal{T}$, but explain how to define the states, the labelling and the transitions. Describe the initial state as well.) (8 marks)

(b) Describe the differences between runs and computations on labelled transition systems. (6 marks)

(c) Explain what deadlock is and show that there is a run σ on $\mathcal{T}$ that does not avoid deadlock. (6 marks)

(d) Define a formula φ of linear temporal logic which shows that σ from the previous item reaches a deadlocked state. (5 marks)

6. Consider the labelled program from the previous question. Let $crit_i$ be the formula $L3_i \vee L4_i \vee L5_i$ (for every $0 \leq i \leq 4$) expressing that process i is in its critical section, i.e., at one of the locations $L3, L4, L5$.

(a) Define (using $crit_i$) formulas *some-crit*, *none-crit* and *one-crit* expressing that "at least one process is in its critical section", "no process is in its critical section" and "precisely one process is in its critical section", respectively. (5 marks)

(b) Show that $(\textit{one-crit} \leftrightarrow s = 0) \wedge (\textit{none-crit} \leftrightarrow s = 1) \wedge (s = 0 \vee s = 1)$ is inductive. (12 marks)

(c) Show that $\textit{some-crit} \rightarrow \textit{one-crit}$ is invariant. (8 marks)

