Birkbeck
(University of London)

MSc and MRes Examination for Internal Students
MSc in Advanced Information Systems
MSc in Web Information Management
MRes in Computer Science

School of Computer Science and Information Systems

Knowledge Representation and Reasoning (COIY027P)

Date of examination: 15/06/2009
Duration of paper: 10:00–12:00

There are 4 questions on this paper. Candidates should attempt all of them. Calculators are not permitted.

1. Consider the following arguments.
ARG1: "If I am clever and I do the homework, then I pass the exam. Therefore, if I am clever, then I pass the exam, or if I do the homework, then I pass the exam."
ARG2: "If I am clever and I do the homework, then I pass the exam. Therefore, if I am clever or I do the homework, then I pass the exam."

   (a) Formalise the above arguments using propositional logic. (5 marks)

   (b) Determine whether the arguments are valid. Justify your answer. (20 marks)

2. A frame is called *dense* if it satisfies the following condition:

$$\forall s \forall t (sRt \to \exists u (sRu \wedge uRt))$$

   i.e., for every pair of related points, there is an intermediate point. Consider the following formula

$$\text{DENSE:} \qquad \Diamond\varphi \to \Diamond\Diamond\varphi.$$

   (a) Prove that the formula DENSE is valid in dense frames. (12 marks)

   (b) Show that the formula DENSE is not valid in general. (8 marks)

   (c) Is DENSE valid in epistemic logic, where $\Diamond\varphi$ stands for $\neg K \neg \varphi$? (5 marks)

3. Consider the following variant of the muddy children puzzle. The father's (true) public announcement is:

   "If at least one of you is muddy, then there are more muddy children than clean children."

   Assume that there are four (intelligent and truthful) children and that the rules of the game are common knowledge.

   (a) Draw a Kripke model representing the children's knowledge after the father's announcement. (5 marks)

   (b) Determine what happens in the first and second rounds when the father repeatedly asks: "Raise your hand if you know whether you are muddy." (10 marks)

   (c) Give a formula φ that is common knowledge among the children *before* the father's announcement, but it is not common knowledge afterwards. (10 marks)

4. Consider the following statements. "Alice has two sons, Bob and Carl. Hence Bob and Carl are brothers (to each other). Dan is the father of both Bob and Carl. Bob is clever and Carl is not."

   (a) Design a knowledge base Σ using the $\mathcal{ALC}$ description logic expressing the above facts. (5 marks)

   (b) Decide whether it semantically follows from Σ that the three concepts below are true of Alice (where $\varphi \to \psi$ abbreviates $\neg\varphi \vee \psi$ and $\varphi \leftrightarrow \psi$ abbreviates $\varphi \to \psi \wedge \psi \to \varphi$ as usual):

$$\Sigma \models \exists \text{hasSon.}(\text{Clever} \to \text{hasBrother.}(\text{Clever} \wedge \exists \text{hasFather.Clever}))(\text{Alice})$$
$$\Sigma \models \exists \text{hasSon.}(\text{hasBrother.}(\text{Clever} \wedge \exists \text{hasFather.Clever}) \to \text{Clever})(\text{Alice})$$
$$\Sigma \models \exists \text{hasSon.}(\text{Clever} \leftrightarrow \text{hasBrother.}(\text{Clever} \wedge \exists \text{hasFather.Clever}))(\text{Alice})$$

   (20 marks)

