Birkbeck
(University of London)

MSc Examination

Department of Computer Science and Information Systems

KNOWLEDGE REPRESENTATION AND REASONING (COIY027H7)

CREDIT VALUE: 15 credits

Date of examination: 28/05/2013
Duration of paper: 10:00–12:00

There are FIVE questions in this paper, each worth 25 marks.
Candidates should answer FOUR questions.
Calculators are not permitted.

1. Consider the following argument. "If I am clever, then I pass the exam. Also, if I did both the homework and the exercise, then I pass the exam. Therefore if I fail the exam, then I am not clever or I failed to do the homework and I failed to do the exercise." Observe that the conclusion is ambiguous.

   (a) Give formalisations in propositional logic of the two readings of the above argument. (10 marks)

   (b) Determine whether the above formalisations are valid. (15 marks)

2. Consider the following formulas

$$\psi_1 : \Box\Diamond\varphi \qquad \psi_2 : \Diamond\Box\varphi$$

   (a) Determine whether $\psi_1 \to \psi_2$ and $\psi_2 \to \psi_1$ are valid in modal logic **S5**. (17 marks)

   (b) Repeat the previous task for modal logic **K**. (8 marks)

3. Consider the temporal logic formulas

$$\varphi_1 : \psi_1 U(\psi_2 \wedge \psi_3) \qquad \varphi_2 : (\psi_1 U \psi_2) \wedge (\psi_1 U \psi_3)$$

   (a) Determine whether $\varphi_1 \wedge \varphi_2$ is satisfied over the following labelled transition system L. There are four states: s_0 (the initial state), s_1, s_2 and s_3. The transitions are $\{(s_0, s_1), (s_0, s_2), (s_1, s_2), (s_1, s_3), (s_2, s_1), (s_2, s_3), (s_3, s_3)\}$. The labelling is given by $\ell(s_0) = \{\psi_1\}$, $\ell(s_1) = \{\psi_1, \psi_2\}$, $\ell(s_2) = \{\psi_3\}$ and $\ell(s_3) = \{\psi_2, \psi_3\}$. (10 marks)

   (b) Determine whether $\varphi_1 \to \varphi_2$ and $\varphi_2 \to \varphi_1$ are valid linear temporal logic formulas. (15 marks)

4. Consider the following concurrent program, also known as *strict alternation*.

$$\textbf{input variable: } t = 1$$

$$P_1 \left[\begin{array}{l} \textbf{while}(\text{true})\{ \\ \ \textbf{noncritical} \\ \ \textbf{while}(t! = 1) \\ \ \textbf{critical} \\ \ t = 2\} \end{array} \right] \quad || \quad P_2 \left[\begin{array}{l} \textbf{while}(\text{true})\{ \\ \ \textbf{noncritical} \\ \ \textbf{while}(t! = 2) \\ \ \textbf{critical} \\ \ t = 1\} \end{array} \right]$$

   (a) Describe a transition system that is a suitable model for the above program. (10 marks)

(b) Define a temporal logic formula expressing mutual exclusion, and determine whether the formula is true during all computations. (15 marks)

5. Consider the following statements. "Alice has two sons, Bob and Carl. Hence Bob and Carl are brothers (of each other). Dan is the father of both Bob and Carl. Bob is clever and Carl is not."

(a) Design a knowledge base Σ using the $\mathcal{ALC}$ description logic expressing the above sentences. (5 marks)

(b) Decide whether it semantically follows from Σ that the three concepts below are true of Alice (where $\varphi \rightarrow \psi$ abbreviates $\neg\varphi \vee \psi$ and $\varphi \leftrightarrow \psi$ abbreviates $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$ as usual):

$$\Sigma \models \exists\text{hasSon}.(\text{Clever} \rightarrow \text{hasBrother}.(\text{Clever} \wedge \exists\text{hasFather}.\text{Clever}))(\text{Alice})$$
$$\Sigma \models \exists\text{hasSon}.(\text{hasBrother}.(\text{Clever} \wedge \exists\text{hasFather}.\text{Clever}) \rightarrow \text{Clever})(\text{Alice})$$
$$\Sigma \models \exists\text{hasSon}.(\text{Clever} \leftrightarrow \text{hasBrother}.(\text{Clever} \wedge \exists\text{hasFather}.\text{Clever}))(\text{Alice})$$

(20 marks)

