**Birkbeck**
**(University of London)**

**MSc and MRes Examination for Internal Students**
***MSc in Advanced Information Systems***
***MSc in Web Information Management***
***MRes in Computer Science***

**School of Computer Science and Information Systems**

# Knowledge Representation and Reasoning (COIY027P)

**Date of examination: 26/05/2006**
**Duration of paper: (14:30–16:30)**

*There are seven questions on this paper. Candidates should attempt any FOUR of them. Question 7 is for the benefit of resit students. Calculators are not permitted.*

 

1. Consider the following argument. "If I did the homework, or I am very clever, then I pass the exam. I pass the exam, but I did not do the homework. Therefore I am very clever."

   (a) Formalise the above argument. (10 marks)

   (b) Determine whether the argument is valid. (15 marks)

2. A frame is called *serial* if, for every possible world, there is at least one possible world it is related to, i.e.,

$$\forall s \exists t (sRt).$$

   Consider the following formula

$$\text{SERIAL:} \qquad \Box\varphi \to \Diamond\varphi.$$

   (a) Prove that the formula SERIAL is valid in serial frames. (15 marks)

   (b) Show that the formula SERIAL is not valid in general. (10 marks)

3. A modal logic formula is called *positive* if it is built up from propositional atoms by using the connectives conjunction and box only (i.e., negation does not occur in the formula). The model $\mathcal{M}' = (S', R', V')$ is called a *submodel* of the model $\mathcal{M} = (S, R, V)$ if $S' \subseteq S$, $R' = R \cap (S' \times S')$ and, for every $p$, $V'(p) = V(p) \cap S'$.

   (a) Show that the truth of positive formulas are preserved in submodels, i.e., for every positive formula $\varphi$ and $s \in S'$,

$$(\mathcal{M}, s) \models \varphi \text{ implies } (\mathcal{M}', s) \models \varphi$$

   whenever $\mathcal{M}'$ is a submodel of $\mathcal{M}$. (15 marks)

   (b) Show that the reverse of the implication above is not true. (10 marks)

4. Consider the following variant of the wise men puzzle. It is common knowledge that there are four wise men and at most two white hats.

   (a) Assume that both the first and second wise men are wearing white hats. Show that it is known by everybody that at least one of them is wearing a white hat ($E(w_1 \vee w_2)$), but it is not common knowledge ($\neg C(w_1 \vee w_2)$). (15 marks)

   (b) Assume that the first wise man answers "No" to the king's question whether he knows the colour of his hat. Show that it is common knowledge that at least one of the third or fourth wise men is wearing a red hat ($C(r_3 \vee r_4)$). (10 marks)

5. Consider the following TBox $\Sigma$:

$$\begin{aligned}
&\text{Mother} \equiv \exists\text{hasChild}.\top \sqcap \text{Female}\\
&\text{Father} \equiv \exists\text{hasChild}.\top \sqcap \text{Male}\\
&\text{Grandmother} \equiv \exists\text{hasChild}.(\text{Mother} \sqcup \text{Father}) \sqcap \text{Female}\\
&\text{Grandfather} \equiv \exists\text{hasChild}.(\text{Mother} \sqcup \text{Father}) \sqcap \text{Male}
\end{aligned}$$

   We define:

$$\text{Grandparent}_1 \equiv \exists\text{hasChild}.(\exists\text{hasChild}.\top) \quad \text{and} \quad \text{Grandparent}_2 \equiv \text{Grandmother} \sqcup \text{Grandfather}$$

   (a) Determine which one of the following is true:

$$\Sigma \models \text{Grandparent}_1 \sqsubseteq \text{Grandparent}_2 \quad \text{or} \quad \Sigma \models \text{Grandparent}_2 \sqsubseteq \text{Grandparent}_1$$

   (15 marks)

 

(b) Add a formula to $\Sigma$ so that the two definitions of Grandparent become equivalent. (10 marks)

6. "Alice has two sons, Bob and Carl. Dan is the father of both Bob and Carl. Bob is clever and Carl is not."

(a) Design a knowledge base $\Sigma$ expressing the above facts. (5 marks)

(b) Decide whether the following are true:

$$\Sigma \models \exists \text{hasSon}.(\text{Clever} \leftrightarrow \exists \text{hasFather}.\text{Clever})(\text{Alice})$$
$$\Sigma \models \exists \text{hasSon}.(\text{Clever} \leftrightarrow \forall \text{hasFather}.\text{Clever})(\text{Alice})$$

(20 marks)

7. Determine whether the following formulas of linear temporal logic are valid:

(a)

$$(\varphi \mathsf{U} \psi) \leftrightarrow (\psi \vee (\varphi \wedge \mathsf{X}(\varphi \mathsf{U} \psi))).$$

(10 marks)

(b)

$$(\psi \mathsf{U}(\varphi \rightarrow \chi) \wedge \psi \mathsf{U} \varphi) \rightarrow \psi \mathsf{U} \chi.$$

(15 marks)