Birkbeck
(University of London)

MSc and MRes Examination for Internal Students
School of Computer Science and Information Systems

Knowledge Representation and Reasoning (COIY027H7)

Date of examination: 21/05/2010
Duration of paper: 14:30–16:30

There are 4 questions in this paper, each is worth 25 marks. Candidates should attempt all of them. Calculators are not permitted.

1. (a) Consider the following argument.

"If I am clever, then I pass the exam, unless I am unlucky. Therefore, if I am both clever and lucky, then I pass the exam."

Formalise the above argument in propositional logic and determine whether it is valid by using the semantic tableau method. (12 marks)

(b) Prove that the following two formulas are semantically equivalent for any natural number n:

$$(a_1 \wedge a_2 \wedge a_3 \wedge \cdots \wedge a_{n-1} \wedge a_n) \rightarrow b$$
$$(a_1 \rightarrow (a_2 \rightarrow (a_3 \rightarrow (\ldots a_{n-1} \rightarrow (a_n \rightarrow b) \ldots)))$$

[Hint: Use induction on n.] (13 marks)

2. Consider the following formulas

$$\psi_1 : \quad \Box\Diamond\varphi$$
$$\psi_2 : \quad \Diamond\Box\varphi$$

(a) Determine whether $\psi_1 \rightarrow \psi_2$ and $\psi_2 \rightarrow \psi_1$ are valid in modal logic **S5**. (17 marks)

(b) Repeat the previous task for modal logic **K**. (8 marks)

3. Assume that there are five (intelligent and truthful) brothers called Monday, Tuesday, Wednesday, Thursday and Friday, respectively, and that the rules of the game are common knowledge. They are standing in a queue so that each of them can see the backs of his brothers in front of him. Thus Friday can see the back of Thursday, Wednesday, Tuesday and Monday, Thursday can see the back of Wednesday, Tuesday and Monday, etc., and Monday cannot see the back of anybody. The father puts a sticker on the back of each of the brothers without revealing the colour of the sticker to him or to his brothers in the queue in front of him (if any), but the colour of the sticker is visible to his brothers in the queue behind him (if any). Then the father makes the following, true announcement (that the brothers take for granted):

"Precisely one of you has a red sticker on his back."

(a) Describe the Kripke model representing the brothers' knowledge after the father's announcement. (8 marks)

(b) Determine what happens in the successive rounds when the father repeatedly asks: "If you know that you are wearing the red sticker, shout 'I am the red one'." (12 marks)

(c) Give a formula φ that is common knowledge among the brothers *before* the first round, but it is not common knowledge afterwards. (5 marks)

4. Consider the following statements. "Bob and Carl are sons of Alice. Bob and Carl are each other's brothers."

(a) Design a knowledge base Σ using the $\mathcal{ALC}$ description logic expressing the above facts. (5 marks)

(b) Decide whether it semantically follows from Σ that the three concepts below are true of Alice (where $\varphi \rightarrow \psi$ abbreviates $\neg\varphi \vee \psi$ as usual):

$$\Sigma \models \exists \text{hasSon}.(\text{Clever} \rightarrow \exists \text{hasBrother}.\text{Clever})(\text{Alice})$$

$$\Sigma \models [(\exists \text{hasSon}.\text{Clever} \wedge \forall \text{hasSon}.(\text{Clever} \rightarrow \exists \text{hasBrother}.\text{Clever})) \rightarrow \exists \text{hasSon}.\exists \text{hasBrother}.\text{Clever}](\text{Alice})$$

$$\Sigma \models (\exists \text{hasSon}.\neg \text{Clever} \rightarrow \forall \text{hasSon}.\exists \text{hasBrother}.\neg \text{Clever})(\text{Alice})$$

(15 marks)

(c) Find three distinct formulas φ_1, φ_2 and φ_3 such that, for any knowledge base Γ containing Σ, we have $\Gamma \models \varphi_1 \wedge \varphi_2 \wedge \varphi_3$. Justify your answer. (5 marks)

