Overview of my most important publications

Igor Razgon

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1 Parameterized Complexity

- *Jianer Chen, Yang Liu, Songjian Lu, Barry O'Sullivan, Igor Razgon: A fixed-parameter algorithm for the directed feedback vertex set problem. J. ACM 55(5): 21:1-21:19 (2008)*

  By the time we obtained the result, the fixed-parameter tractability of the directed feedback vertex set was considered one of the most challenging open questions in the area of parameterized complexity. Last year the paper was warded Nerode Prize.

- *Dániel Marx, Barry O'Sullivan, Igor Razgon: Finding small separators in linear time via treewidth reduction. ACM Trans. Algorithms 9(4): 30:1-30:35 (2013)*

  We proved so-called the Treewidth Reduction Theorem. Using the theorem we resolved several seemingly unrelated open problems in the area of parameterized complexity. Since publication of our result, the theorem became a well known tool for design of fixed-parameter algorithms for graph separation problems. Interestingly that we recently used the result design of FPT algorithms for problems on *hypergraphs* rather than graphs, please see Section 4 for further details.

2 Knowledge Compilation

- *Igor Razgon, "On the Read-Once Property of Branching Programs and CNFs of Bounded Treewidth". Algorithmica 75(2): 277-294 (2016)*

The paper demonstrates that there is a class of CNFs of bounded (primal) treewidth k that requires XP size ($n^{O(k)}$) nondeterminisitc read-once branching programs (NROBP) to represent them.

NROBPs generalize such important models as Ordered Binary Decision Diagrams (OBDDs) and Free Binary Decision Diagrams (FBDDs). Therefore, the paper shows that these models are not good for representation of CNFs of bounded treewidth and stronger models must be considered.

From the perspective of knowledge compilation, the significance of this result is that it provides a 'watershed' between many models based on Decomposable Negation Normal Forms (DNNFs) that have FPT sized representation of CNFs of bounded treewidth and syntactic read-once branching programs. In particular, several quasipolynomial simulations of a DNNF model by a read-once branching program are known to be essentially tight because of the above result.

The above result gave rise to several directions of further research. A progress in these directions have been recently obtained in (Razgon, IPEC21) and (Razgon, arXiv:2201.02173).

3 Order Theory

- *Vadim V. Lozin, Igor Razgon, Viktor Zamaraev: Well-quasi-ordering versus clique-width. J. Comb. Theory, Ser. B 130: 1-18 (2018)*

A famous result in graph theory is the Graph Minor Theorem of Robertson and Seymour stating that the graph minor relation defined on the set of all graph is well quasi ordered (WQO). This result led researchers to consider other order relations on graphs. One such a natural relation is induced subgraphs. These relation can be easily shown to be not WQO. However many restricted (hereditary) classes of graphs did turn out to be WQO by the induced sugraph relation. At some point it has been observed that all such known classes had a constant cliquewidth. A natural question was raised as to whether this is the case in general. This question stood open for about five years despite a significant attention from researchers in the area.

In this paper we resolved the above question negatively by demonstrating a hereditary class of graphs that is WQO by the induced subgraphs relation and yet has unbounded cliquewidth.

3.1 Hypertree Width

- *Georg Gottlob, Matthias Lanzinger, Reinhard Pichler, Igor Razgon: Fractional Covers of Hypergraphs with Bounded Multi-Intersection. MFCS 2020: 41:1-41:14*

In this paper we proved polynomial computability of the fractional hypertree width for hypergraphs of *bounded multiple intersection* (whose incidence graphs do not contain large bicliques). The most important part of the proof is a statement that may be of an independent interest in the area of polyhedral combinatorics.

Consider a hypergraph H with n vertices and m edges and let $\mathbf{A}$ be the $n \times m$ incidence matrix of H. Let $\mathbf{x} = (x_1, \ldots, x_m)$ be a vector, fix a constant k and consider a polytope $P(H) = \{\mathbf{x} | \sum_i x_i \leq k, \mathbf{Ax} \geq \mathbf{1}, \forall_i 0 \leq x_i \leq 1\}$ where $\mathbf{1}$ is the vector of n components all equal 1.

Denote by $support_k(H)$ the smallest possible number of positive components of $\mathbf{x} \in P(H)$. How small $support_k(H)$ may be? If we assume that $\mathbf{x}$ is integral,

the answer is obvious: at most k edges can (and in fact must) be non-zero. However, without the integrality requirement, the weight can be spread across many hyperedges. As a result, there are classes of hypergraphs H for which $support_k(H)$ *cannot* be upper bounded by *any* function on k.

Our main combinatorial statement is that there is a function h such that $support_k(H) \leq h(k, q)$ where $q = q(H)$ is the smallest number such that the incidence graph of H does not have $K_{q,q}$ as a subgraph. In other words, n and m can be unbounded but, as long as $q(H)$ is small, $support_k(H)$ is small as well.

The result described above, along with our Journal of the ACM paper provide an algorithmic framework for efficient computation of hypertree width parameters for hypergraphs of bounded multiple intersection. However, the algorithms resulting from this framework are all XP. Therefore, the possibility of FPT computation is a natural direction of further investigation. The initial progress in this direction has been reported in (Razgon, 2212.13423).