Logic-based Ontology Comparison and Module Extraction in OWL 2 QL

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joint work with

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Large-scale ontologies

- Life-sciences, healthcare, and other knowledge intensive areas depend on having a **common language** for gathering and sharing knowledge
- Such a common language is provided by **reference terminologies**
- Examples:
  - SNOMED CT (Systematized Nomenclature of Medicine Clinical Terms)
  - NCI (National Cancer Institute Ontology)
  - FMA (Foundational Model of Anatomy)
  - GALEN
  - . . .
- Typical size: at least **50,000** terms and axioms
- Trend towards axiomatising reference terminologies in **('lightweight') description logics**

Description logic $\mathcal{ALCHIQ}$

Vocabulary:

- individuals $a_0, a_1, \ldots$
  (e.g., john, mary) (nominals in ML/constants in FO)
- concept names $A_0, A_1, \ldots$
  (e.g., Person, Female) (variables in ML/unary predicates in FO)
- role names $R_0, R_1, \ldots$
  (e.g., hasChild, loves) (modalities in ML/binary predicates in FO)

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  $a_i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
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- roles

$$R \quad ::= \quad R_i \quad | \quad R_i^-$$

$$(R_i^-)^{\mathcal{I}} = \{(y, x) \mid (x, y) \in R_i^{\mathcal{I}}\}$$

- concepts

$$C \quad ::= \quad A_i \mid \neg C \mid C_1 \sqcap C_2 \quad | \quad \exists R.C \quad | \quad \forall R.C \quad | \quad \geq q R.C$$

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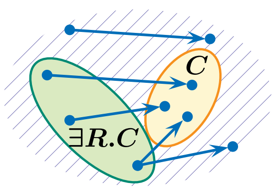

$\forall R.C$ $|||$ $\neg(\exists R.\neg C)$

'there are at least q distinct R-successors that are in C'

Description logic $\mathcal{ALCHIQ}$ (cont.)

knowledge base $\mathcal{K}$ = TBox $\mathcal{T}$ + ABox $\mathcal{A}$

- $\mathcal{T}$ is a set of **terminological axioms** of the form $C \sqsubseteq D$ and $R \sqsubseteq S$
- $\mathcal{A}$ is a set of **assertional axioms** of the form $C(a)$ and $R(a, b)$

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Reasoning:

– satisfiability $\mathcal{K}$

is there a *model* $\mathcal{I}$ for $\mathcal{K}$ $(\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}})$
$(\mathcal{I} \models R \sqsubseteq S$ iff $R^{\mathcal{I}} \subseteq S^{\mathcal{I}})$

– subsumption $\mathcal{K} \models C \sqsubseteq D$

$\mathcal{I} \models C \sqsubseteq D$, for each $\mathcal{I}$ with $\mathcal{I} \models \mathcal{K}$

– instance checking $\mathcal{K} \models C(a)$

$a^{\mathcal{I}} \in C^{\mathcal{I}}$, for each $\mathcal{I}$ with $\mathcal{I} \models \mathcal{K}$

– query answering $\mathcal{K} \models q(\vec{a})$, $q(\vec{a})$ a positive existential formula

$\mathcal{I} \models q(a)$ (as a first-order structure), for each $\mathcal{I}$ with $\mathcal{I} \models \mathcal{K}$

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OWL 1.0 DL is based on $\mathcal{SHOIQ}(D)$, **OWL 2.0** on $\mathcal{SROIQ}(D)$

$\mathcal{ALCHIQ}$ + transitive roles + nominals + concrete domains

$\mathcal{SHOIQ}(D)$ + role chains + disjoint roles + self (diagonal)

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- **module extraction:**
  computing a subset $\mathcal{M}$ (ideally as small as possible) of an ontology $\mathcal{T}$ that
  'says' the same about Σ as $\mathcal{T}$

new types of reasoning problems

Σ-Inseparability

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be TBoxes and Σ a **signature** (concept and role names)

When do $\mathcal{T}_1$ and $\mathcal{T}_2$ 'say' the same about Σ?

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- $\mathcal{T}_1$ and $\mathcal{T}_2$ are **Σ-query inseparable** if, for all Σ-queries $q(\vec{x})$ and ABoxes $\mathcal{A}$,

  $\mathcal{T}_1 \equiv^q_\Sigma \mathcal{T}_2$ $(\mathcal{T}_1, \mathcal{A}) \models q(\vec{a})$ iff $(\mathcal{T}_2, \mathcal{A}) \models q(\vec{a})$, for all $\vec{a}$

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- $\mathcal{T}_1$ and $\mathcal{T}_2$ are **Σ-model inseparable** if, for all Σ-interpretations $\mathcal{I}$,

  $\mathcal{T}_1 \equiv^m_\Sigma \mathcal{T}_2$ $\qquad$ $\exists\, \mathcal{I}_1 \supseteq \mathcal{I} \;\; \mathcal{I}_1 \models \mathcal{T}_1$ iff $\exists\, \mathcal{I}_2 \supseteq \mathcal{I} \;\; \mathcal{I}_2 \models \mathcal{T}_2$

Examples

Example 1. $\Sigma = \{\text{Lecturer}, \text{Course}\}$

$$\mathcal{T}_1 = \emptyset, \qquad \mathcal{T}_2 = \{\text{Lecturer} \sqsubseteq \exists\text{teaches}, \ \exists\text{teaches}^- \sqsubseteq \text{Course}\}$$

- Is $\mathcal{T}_1 \equiv^c_\Sigma \mathcal{T}_2$?
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Example 3. $\Sigma = \{A\}$

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Why OWL 2 QL?

- [GLW06] concept inseparability in $\mathcal{ALC}$ is 2ExpTime-complete
- [LWW07] concept inseparability in $\mathcal{ALCQI}$ is 2ExpTime-complete
  in $\mathcal{ALCQIO}$ is undecidable
- [LW07] model inseparability in $\mathcal{EL}$ is undecidable
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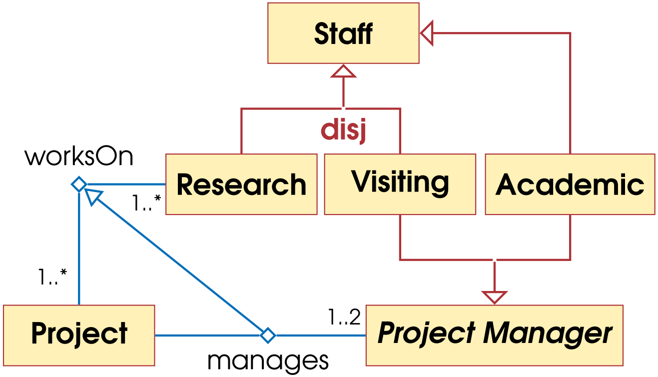
OWL 2 QL is a W3C standard language for OBDA

OWL 2 QL

OWL 2 QL represents inclusions between 1-ary predicates (**concepts**) and the domains and ranges of 2-ary predicates (**roles**), as in ER data models



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$\text{Academic} \sqsubseteq \text{Staff}$

$\exists \text{manages}.\top \sqsubseteq \text{ProjectManager}$

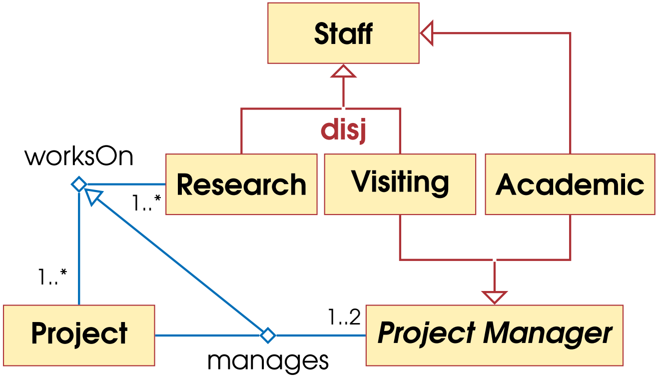
$\exists \text{manages}^-.\top \sqsubseteq \text{Project}$

$\text{Project} \sqsubseteq \exists \text{manages}^-.\top$

$\text{manages} \sqsubseteq \text{worksOn}$

$\geq \mathbf{3}\ \text{manages}^-.\top \sqsubseteq \bot$

$\text{ProjectManager} \sqsubseteq \text{Academic} \sqcup \text{Visiting}$



DL-Lite$^{\mathcal{H}}_{\text{core}}$ and Canonical Models

$$R \quad = \quad P \quad | \quad P^- \qquad\qquad B \quad = \quad \bot \quad | \quad \top \quad | \quad A \quad | \quad \exists R$$

$$B_1 \sqsubseteq B_2 \qquad B_1 \sqcap B_2 \sqsubseteq \bot \qquad R_1 \sqsubseteq R_2 \qquad R_1 \sqcap R_2 \sqsubseteq \bot$$

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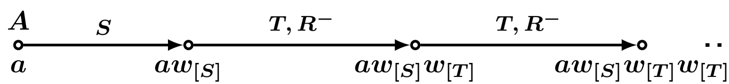
Ex.: $\mathcal{T} = \{A \sqsubseteq \exists S,\ \ \exists S^- \sqsubseteq \exists T,\ \ \exists R \sqsubseteq \exists T,\ \ T \sqsubseteq R^-\}$ and $\mathcal{K} = (\mathcal{T}, \{A(a)\})$

canonical model $\mathcal{M}_{\mathcal{K}}$:



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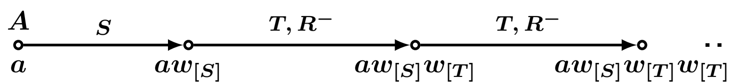

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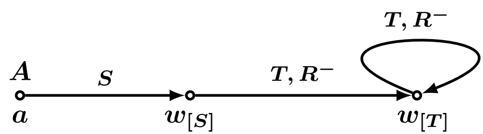
$= \underbrace{\mathsf{tail}}_{\text{the last element}}(\mathcal{M}_{\mathcal{K}})$:



$DL\text{-}Lite^{\mathcal{H}}_{core}$ and Canonical Models

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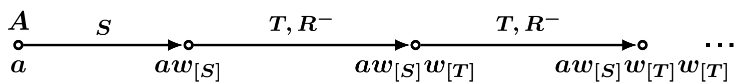

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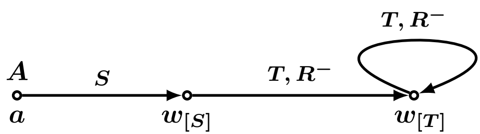

a generates **witnesses** $w_{[S]}$ and $w_{[T]}$: $\qquad a \rightsquigarrow w_{[S]} \rightsquigarrow w_{[T]}$

- $a \rightsquigarrow w_{[S]}$ if $[S]$ is minimal, $\mathcal{K} \models \exists S(a)$ and $\mathcal{K} \not\models S(a,b)$, for all b
- $w_{[S]} \rightsquigarrow w_{[T]}$ if $[T]$ is minimal, $\mathcal{T} \models \exists S^- \sqsubseteq \exists T$ and $[S^-] \neq [T]$

Σ-Query Entailment and Homomorphisms

queries = conjunctive queries (CQs)

Theorem $\mathcal{K} \models \boldsymbol{q} \quad \Leftrightarrow \quad \mathcal{M}_{\mathcal{K}} \models \boldsymbol{q}$, for all consistent $\mathcal{K}$ and all CQ $\boldsymbol{q}$

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NLogSpace

Complexity of Σ-query Entailment

Theorem Checking $\boldsymbol{\Sigma}$-query entailment is **PSpace**-hard

Proof sketch: consider a QBF $\forall \boldsymbol{X_1} \exists \boldsymbol{X_2} \forall \boldsymbol{X_3} \exists \boldsymbol{X_4} \, ((\neg \boldsymbol{X_1} \vee \boldsymbol{X_2}) \wedge \boldsymbol{X_3})$

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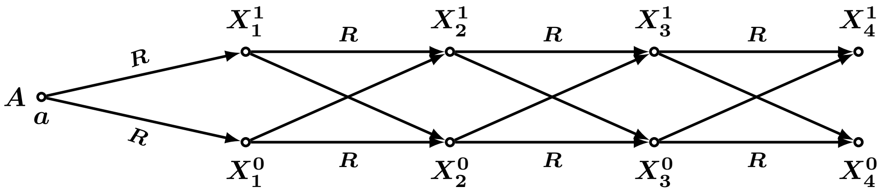

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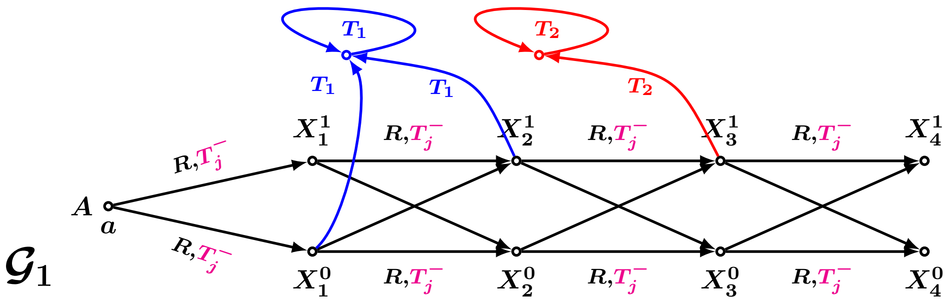

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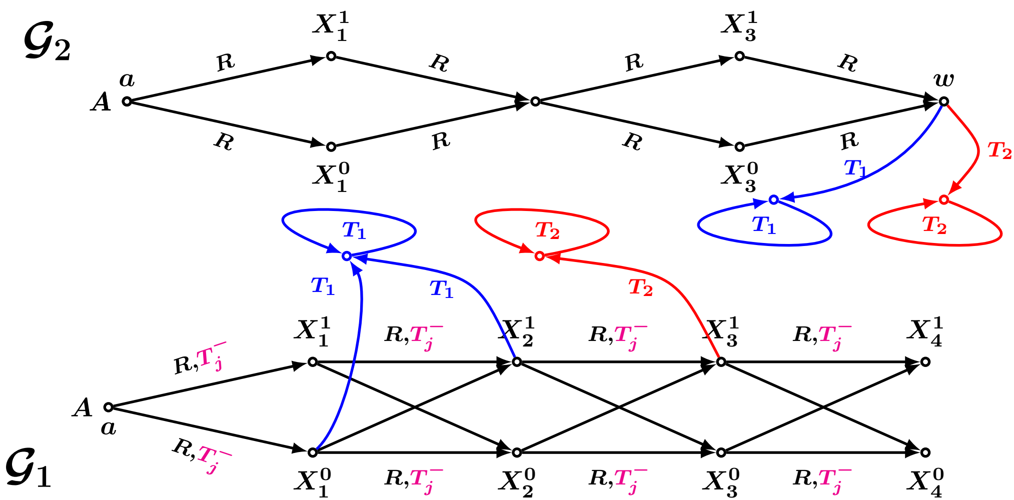

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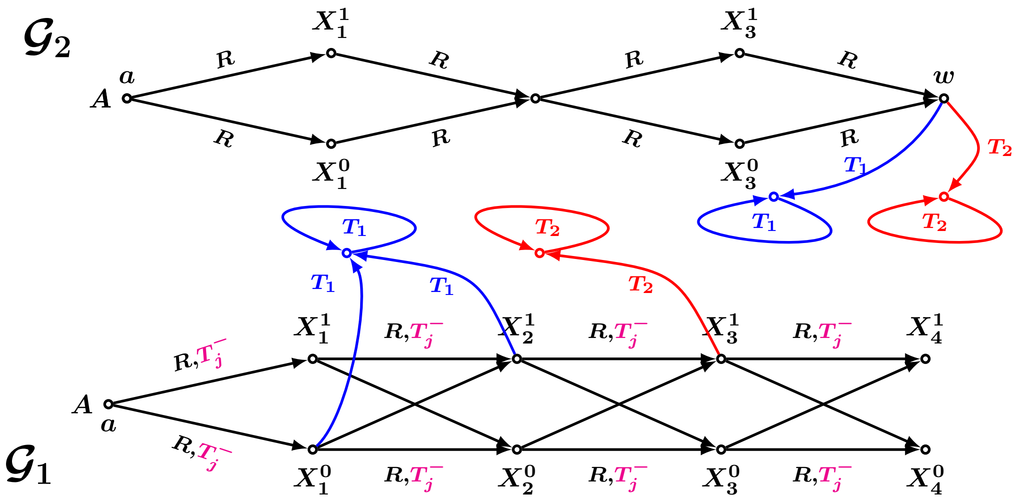

Theorem Checking Σ-query entailment is in **ExpTime**

(alternating 2-way automata)

Polynomial (Incomplete) Algorithms

$\mathcal{T}_1$ Σ-query entails $\mathcal{T}_2$	$\geq$	$\mathcal{T}_1$ Σ-concept/role entails $\mathcal{T}_2$	+	there is a Σ-simulation of $\mathcal{G}_{(\mathcal{T}_2,\{B(a)\})}$ in $\mathcal{G}_{(\mathcal{T}_1,\{B(a)\})}$, for all $\mathcal{T}_1$-consistent Σ-concepts B

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Theorem Without role inclusions, Σ-query entailment is **NLogSpace**-complete

Strong Query Entailment

$\mathcal{T}_1$ and $\mathcal{T}_2$ are **strongly Σ-query inseparable** if, for all Σ-TBoxes $\mathcal{T}$,

$$\mathcal{T}_1 \equiv_{\Sigma}^{sq} \mathcal{T}_2 \qquad \mathcal{T}_1 \cup \mathcal{T} \equiv_{\Sigma}^{q} \mathcal{T}_2 \cup \mathcal{T}$$

exponentially many ($2^{|\Sigma|^2}$) TBoxes $\mathcal{T}$

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more subtle (use the form of OWL 2 QL axioms)

$\mathcal{T}_1$ stronlgly Σ-query entails $\mathcal{T}_2$ = $\mathcal{T}_1$ Σ-query entails $\mathcal{T}_2$ + for each Σ-TBox $\mathcal{T}$ with a **single** $B_1 \sqsubseteq B_2$ or $R_1 \sqsubseteq R_2$, B is $\mathcal{T}_2 \cup \mathcal{T}$-**consistent**, for all $\mathcal{T}_1 \cup \mathcal{T}$-consistent Σ-concepts B

NLogSpace

What is a Module?

Let $\mathcal{S}$ be an inseparability relation, $\mathcal{T}$ a TBox and Σ a signature.

$\mathcal{M} \subseteq \mathcal{T}$ is (a **minimal module of $\mathcal{T}$** cannot be made smaller)

- an **S_Σ-module of $\mathcal{T}$** if $\mathcal{M} \equiv^{\mathcal{S}}_{\Sigma} \mathcal{T}$

- a **depleting S_Σ-module of $\mathcal{T}$** if $\emptyset \equiv^{\mathcal{S}}_{\Sigma \cup \mathbf{sig}(\mathcal{M})} \mathcal{T} \setminus \mathcal{M}$

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  - depleting $\equiv^{q}_{\Sigma}$-module $\Rightarrow$ $\equiv^{q}_{\Sigma}$-module

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  - minimal module extraction algorithm runs in $\mathcal{O}(|\mathcal{T}|^2)$

but the simulation check is **complete**

Module Extraction Algorithms

- minimal S_Σ-module

```
input 𝒯, Σ
ℳ := 𝒯
for each α ∈ ℳ do
  if ℳ \ {α} ≡^S_Σ ℳ then ℳ := ℳ \ {α}
end for
output ℳ
```

NB: depends on the order of axioms in $\mathcal{T}$

Module Extraction Algorithms

- minimal $\mathcal{S}_\Sigma$-module

```
input T, Σ
M := T
for each α ∈ M do
  if M \ {α} ≡^S_Σ M then M := M \ {α}
end for
output M
```

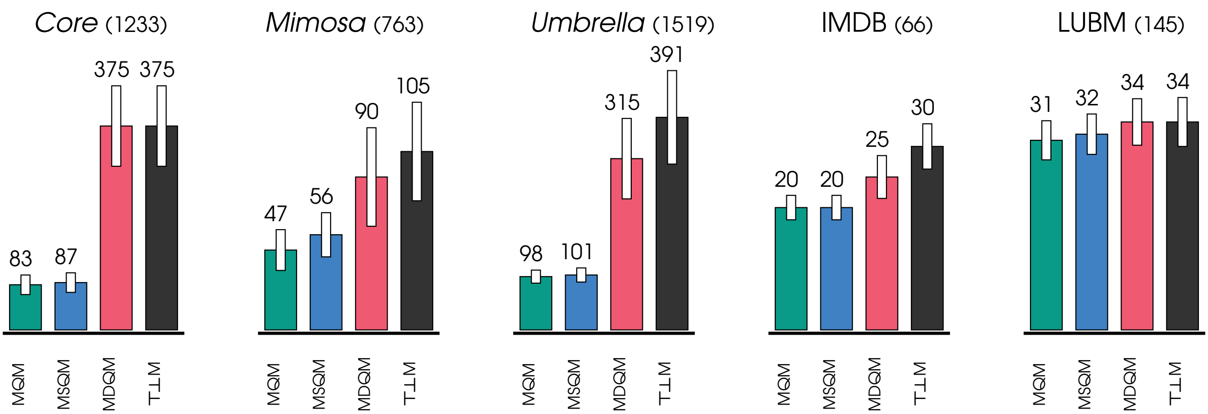
NB: depends on the order of axioms in $\mathcal{T}$

- minimal depleting $\mathcal{S}_\Sigma$-module

```
input T, Σ
T' := T;  Γ := Σ;  W := ∅
while T' \ W ≠ ∅ do
  choose α ∈ T' \ W
  W := W ∪ {α}
  if W ≢^S_Γ ∅ then
    T' := T' \ {α};  W := ∅;  Γ := Γ ∪ sig(α)
  endif
end while
output T \ T'
```

Practical Minimal Module Extraction

MQM = Minimal Query inseparability Module
MSQM = Minimal Strong Query inseparability Module
MDQM = Minimal Depleting Query inseparability Module



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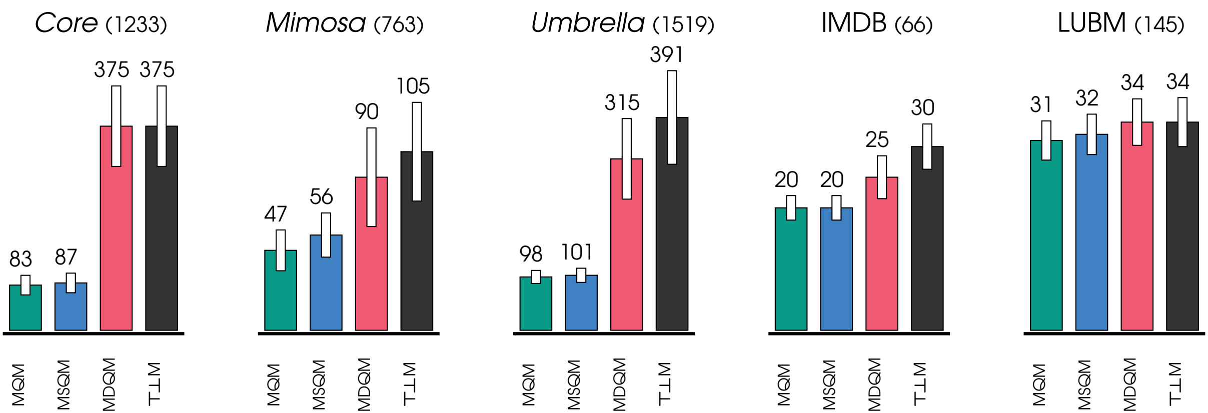

checking query inseparability < 1 sec
checking strong query inseparability < 1 min

only in **9** out of 75,000 query entailment checks did not give a definitive answer due to incompleteness

Σ-inseparability for *DL-Lite*$^{\mathcal{N}}_{bool}$

$$
\begin{array}{lllllllllll}
B & ::= & \bot & \mid & A_i & \mid & \exists R & \mid & \geq q\, R & & \\
C & ::= & B & \mid & \neg C & \mid & C_1 \sqcap C_2 & \mid & C_1 \sqcup C_2 & &
\end{array}
$$

strong Σ-query inseparability $\Leftrightarrow$ Σ-query inseparability

$\Leftrightarrow$ strong Σ-concept inseparability $\Rightarrow$ Σ-concept inseparability

- in each case, the problem is Π_2^p-complete
- can be encoded by Quantified Boolean Formulas $\forall\exists\ \psi$
- modules extracted by QBF solvers

R. Kontchakov, L. Pulina, U. Sattler, T. Schneider, P. Selmer, F. Wolter and M. Zakharyaschev. *Minimal Module Extraction from DL-Lite Ontologies using QBF Solvers.* In C. Boutilier, editor, Proceedings of IJCAI-09 (Pasadena, July 11-17), pp. 836–841, 2009

Example

Let $\mathcal{T}_1$ contain the axioms

$$\begin{array}{llll} \text{Research} \sqsubseteq \exists \text{worksIn}, & \exists \text{worksIn}^- \sqsubseteq \text{Project}, \\ \text{Project} \sqsubseteq \exists \text{manages}^-, & \exists \text{manages} \sqsubseteq \text{Academic} \sqcup \text{Visiting}, \\ \exists \text{teaches} \sqsubseteq \text{Academic} \sqcup \text{Research}, & \text{Academic} \sqsubseteq \exists \text{teaches} \sqcap \leq \mathbf{1}\, \text{teaches}, \\ \text{Research} \sqcap \text{Visiting} \sqsubseteq \bot, & \exists \text{writes} \sqsubseteq \text{Academic} \sqcup \text{Research}, \end{array}$$

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  $q = \exists x\ ((\exists\text{teaches})(x) \wedge (\leq 1\ \text{teaches})(x))$

  'is there anybody who teaches precisely one module?'

$(\mathcal{T}_1, \mathcal{A}) \not\models q \quad (\mathcal{I} \models (\mathcal{T}_1, \mathcal{A})$ but $\mathcal{I} \not\models q)$

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Conclusions

- despite its PSpace-hardness, (strong) Σ-query inseparability can be decided efficiently for real-world OWL 2 QL ontologies
- can our techniques be extended to more expressive DLs such as *DL-Lite*$_{horn}$ or even $\mathcal{ELI}$?
- how can these algorithms be utilised for analysing and visualising the difference between ontology versions?